CHARTING THE PATH
TOWARD THE FUTURE
6th INTERNATIONAL CONFERENCE
Geotechnical Engineering **Education**


# Teaching numerical methods or teaching with numerical methods?

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ICEA

SIMSG ISSMGE

UNIVERSITÉ DE LORRAINE

ÉCOLE NATIONALE SUPÉRIEURE GÉOLOGIE NANCY

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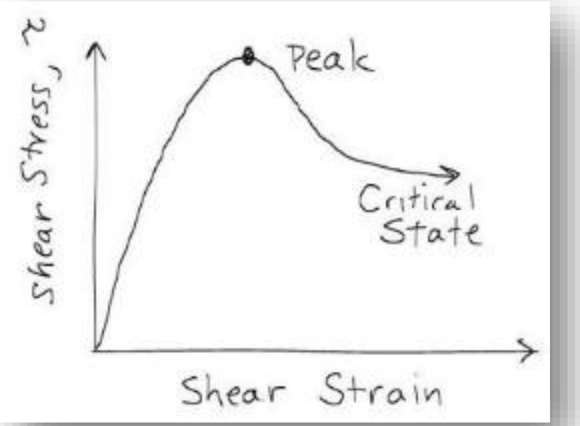
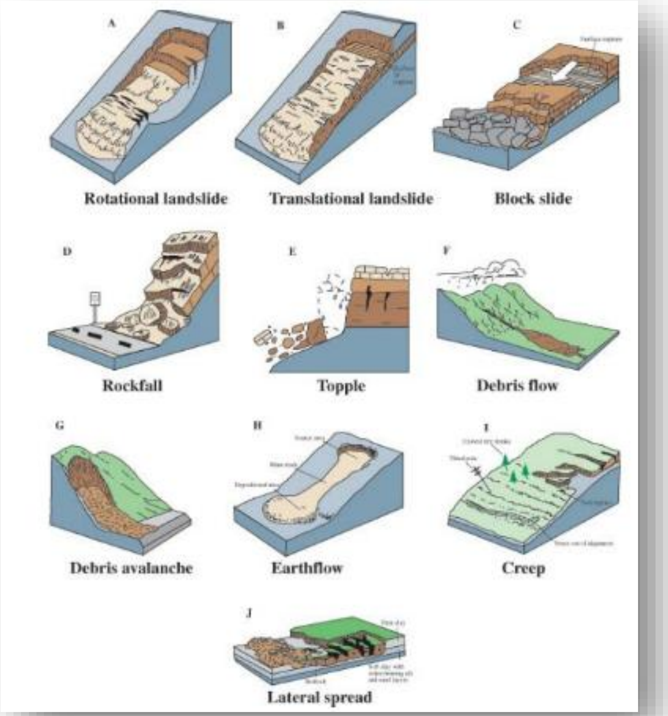
COMITÉ FRANÇAIS DE MÉCANIQUE DES SOLS ET DE GÉOTECHNIQUE

# Introduction

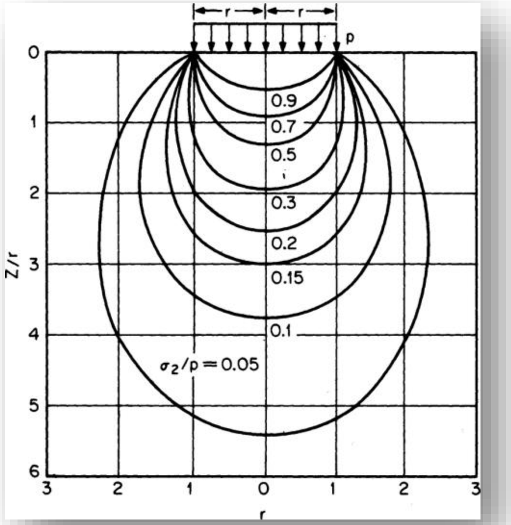
Figures, charts, and videos are fundamental in geotechnical engineering education.



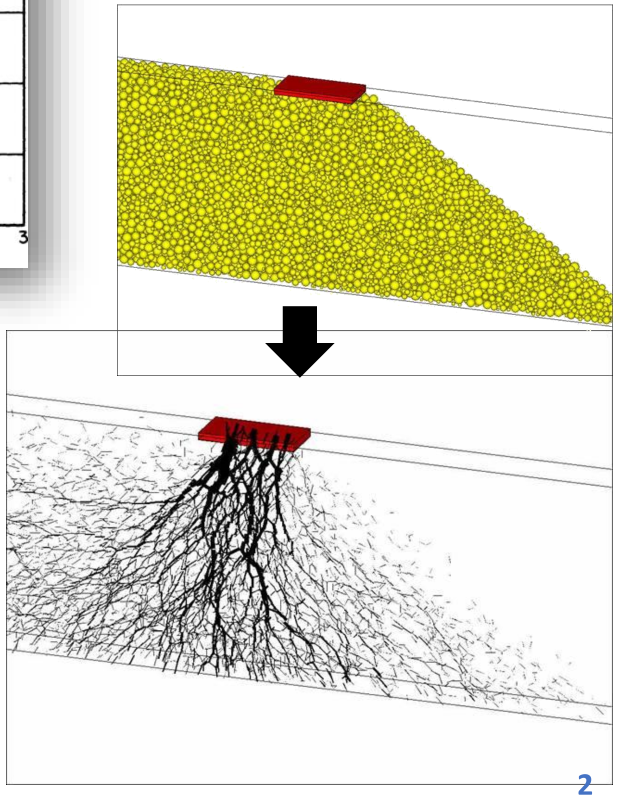

A narrative line graph (Bruce Kutter 2010)




A hybrid, classificational-analytic-narrative set of block diagrams (USGS 2004)




Analytic line graph

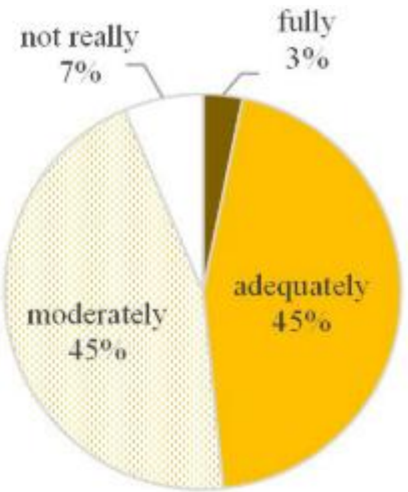


Symbolic 3D diagrams (Gabrieli 2014)

# Introduction

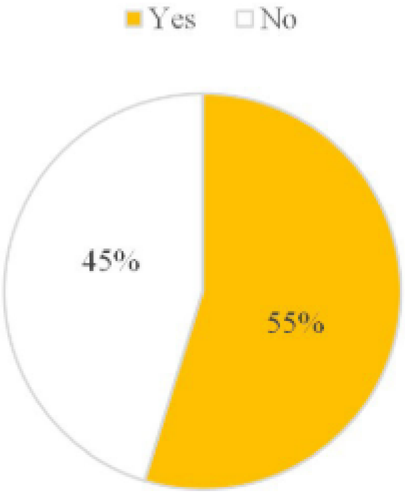
Need for **high-quality** educational materials!

*Are you satisfied with the educational material you currently use in your teaching?*




52% not satisfied with the materials they currently used!

*Were you satisfied with any material found (after searching)?*




***"This finding contradicts the –largely unexamined– popular belief that there is no scarcity of quality educational material at the university level"***


Pantazidou, Marina, and Michele Calvello. 2023. "What Kinds of Educational Material Are Useful for and Desired by University Instructors? The Case of Geotechnical Engineering." *Soils and Rocks* 47 (2): e2024003623. https://doi.org/10.28927/sr.2024.003623.

# Collaboration between TC103 and TC306

**TC306**
**Geoengineering education**

**TC103**
**Numerical methods**

**Objective**: Develop peer-reviewed, high-quality, sharable educational materials using numerical methods

# Fundamental Question

**Can we use numerical methods in education without teaching the underlying mathematical details?**

# Internal survey on ready-to-use material

- Some people have material, but not ready to be shared

→

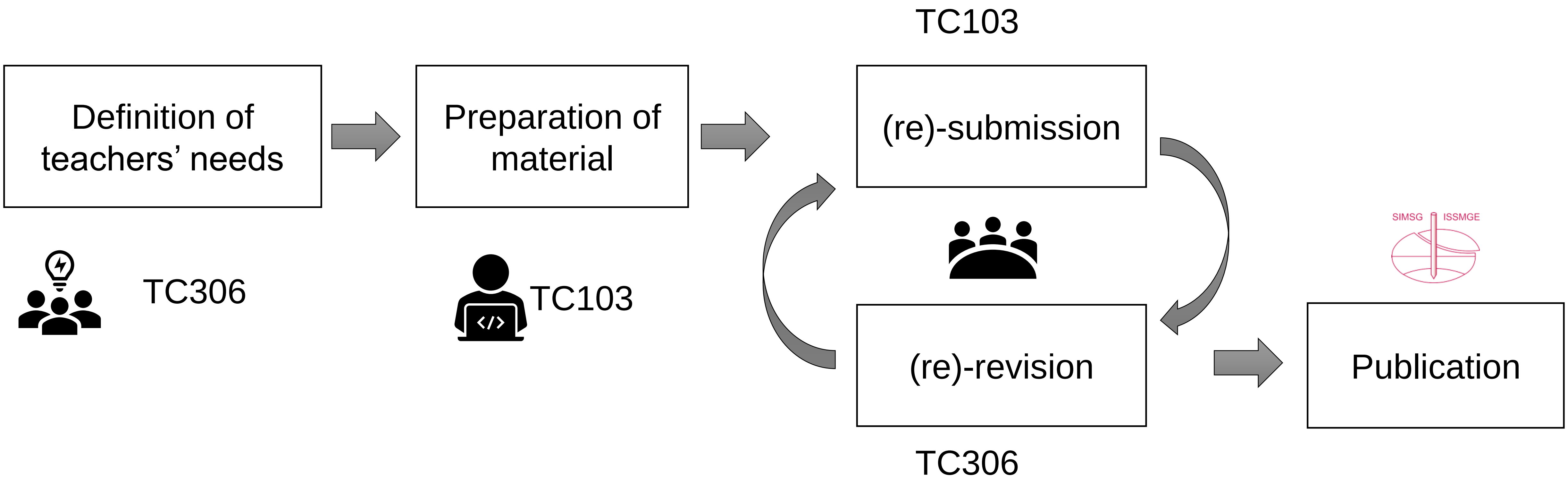
- The «solitude of teaching».
- Necessity to create and engaged comunity!

- Most of the material (from TC103) is for advaced courses

→

- Necessity to create material useful for both basic and advanced courses

# Methodology



# Defining Teachers' Needs

1. Identify the specific topic or concept

   "*Stress distribution under loaded areas*"

2. Explain the current challenge or gap

   "*Students need to be convinced that the theory of elasticity, despite its simplifying assumptions, can provide reasonably accurate results*“

3. Desired learning outcome

   “*Visualize and compare the stress distributions calculated using the theory of elasticity with those obtained using more advanced models suitable for soil behaviour*”

4. Context or examples

5. Potential formats or approaches

***Stress distributions underneath loaded areas***
*“Convince my students and myself that the theory of elasticity performs well: I would like to see how the results would differ if the stress distribution was calculated with something more suitable for soil than the theory of elasticity. I want to see with my own eyes and show students that the difference is indeed small. Also, I would like to provide examples of when the differences are not negligible.“* (M. Pantazidou 2024)

# Preparing Educational Material

- **Product**: figure(s), video(s), Excel sheets, models, ppt slides…
- **Metadata**: should guide the use of the material

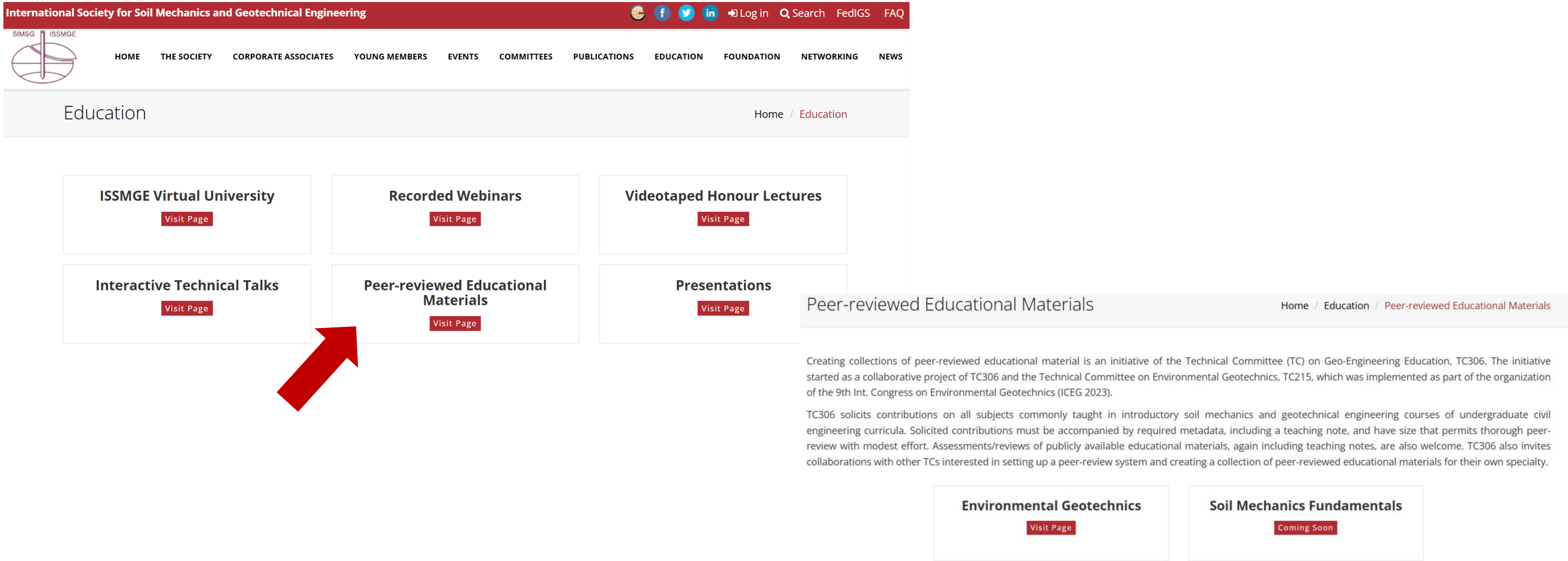
  - Title,
  - Soil mechanics issue/concept,
  - Course chapter/topic,
  - Material type,
  - Brief articulation of main idea,
  - Name and contact of the author,
  - **Additional material** (including detailed explanation of figures and suggestions for instructors)
  - License of use.

# Metadata

- Title,
- Soil mechanics issue/concept,
- Course chapter/topic,
- Material type,
- Name and contact of the author,
- License of use,
- *Educational challenge or gap addressed*
- *Learning outcome*
- *Suggestion for basic and advanced courses*
- Additional material

# Review and Publication

- Peer-review process
- Publication on ISSMGE website (https://www.issmge.org/education/peer-reviewed-educational-materials )



# Results: products

https://researchdata.cab.unipd.it/id/eprint/1549

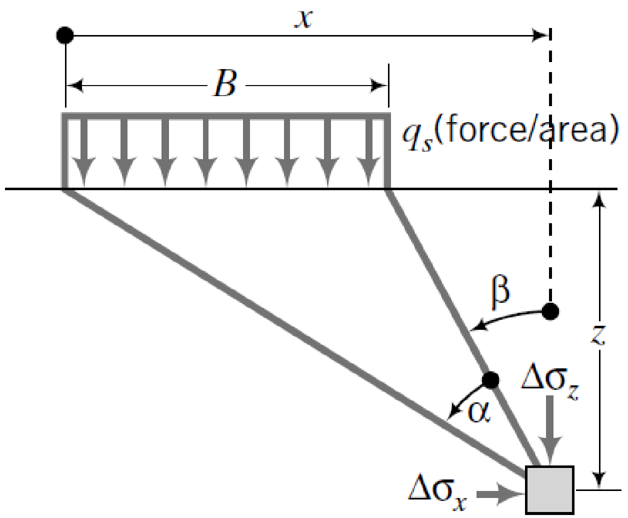
# Shared educational material

- **Contour figures** of normalized stress increments ($\Delta\sigma_z/q$, $\Delta\sigma_x/q$,$\Delta\tau_{zx}/q$) with normalized coordinates (X/B, Z/B) (.fig and .jpg format) obtained with analytical solution.
- **Excel sheets** with normalized stress increments ($\Delta\sigma_z/q$, $\Delta\sigma_x/q$,$\Delta\tau_{zx}/q$) with normalized coordinates (X/B, Z/B).
- **Contour figures** of vertical stresses and displacement obtained with **FEM** using different material parameters and constitutive models.
- **PowerPoint slides** with the analytical and numerical solutions.
- **FEM models** (GeoStudio).

# Analytical solution

Contour figures and Excel file are obtained implementing the analytical solution in Matlab.

### Stress under a strip load




$$\Delta\sigma_z = \frac{q}{\pi}[\alpha + \sin\alpha\cos(\alpha + 2\beta)]$$

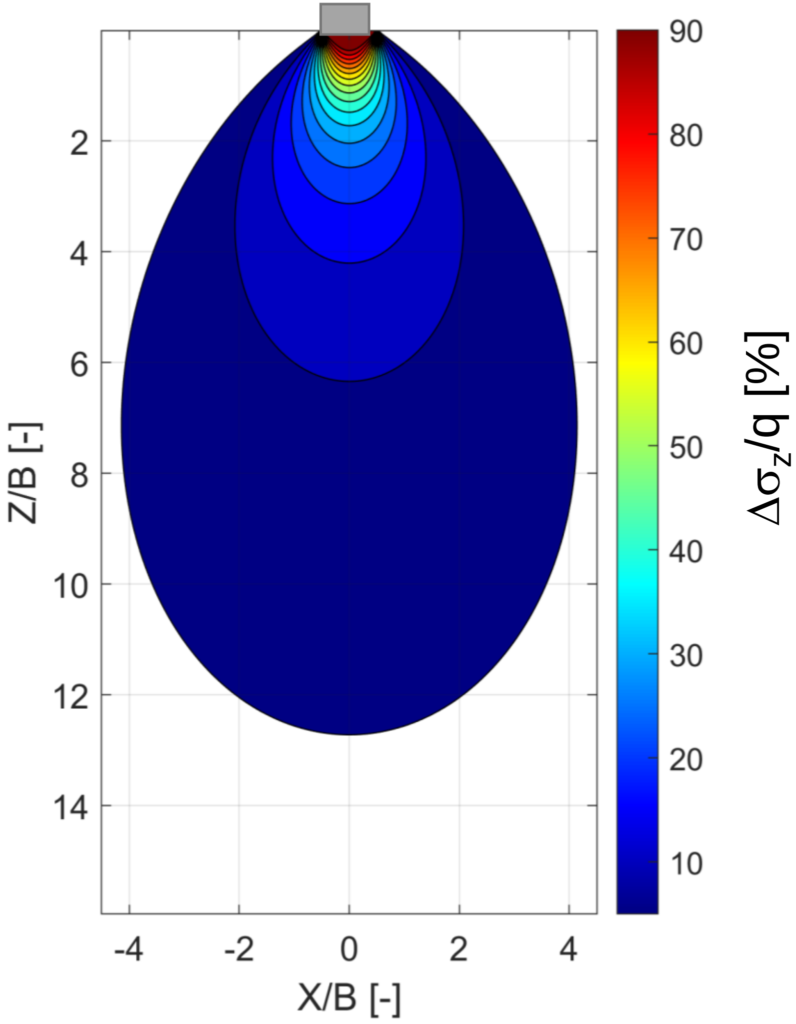
$$\Delta\sigma_x = \frac{q}{\pi}[\alpha - \sin\alpha\cos(\alpha + 2\beta)]$$

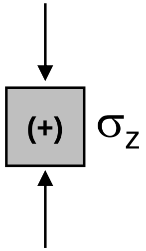
$$\Delta\tau_{zx} = \frac{q}{\pi}[\sin\alpha\cos(\alpha + 2\beta)]$$

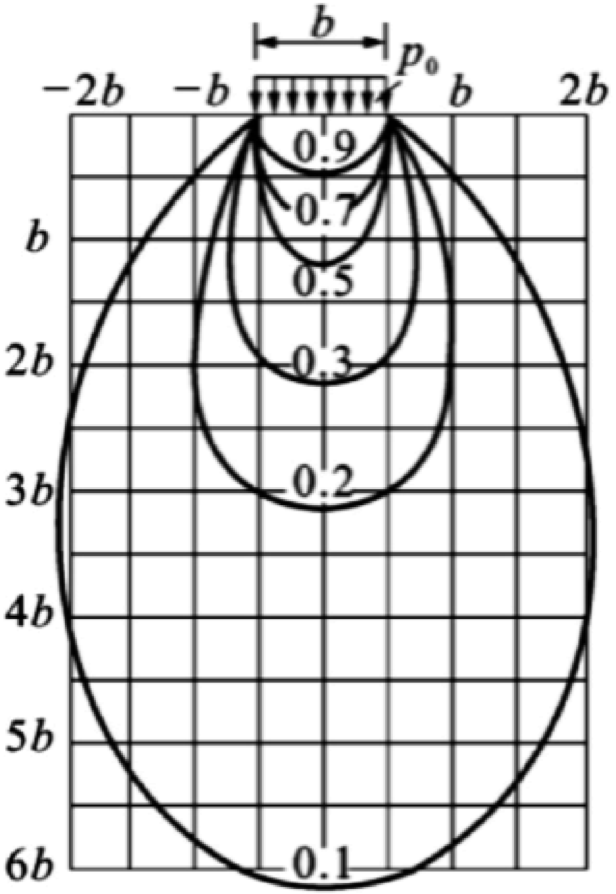
MATLAB

Hypothesis: homogeneous elastic halfspace

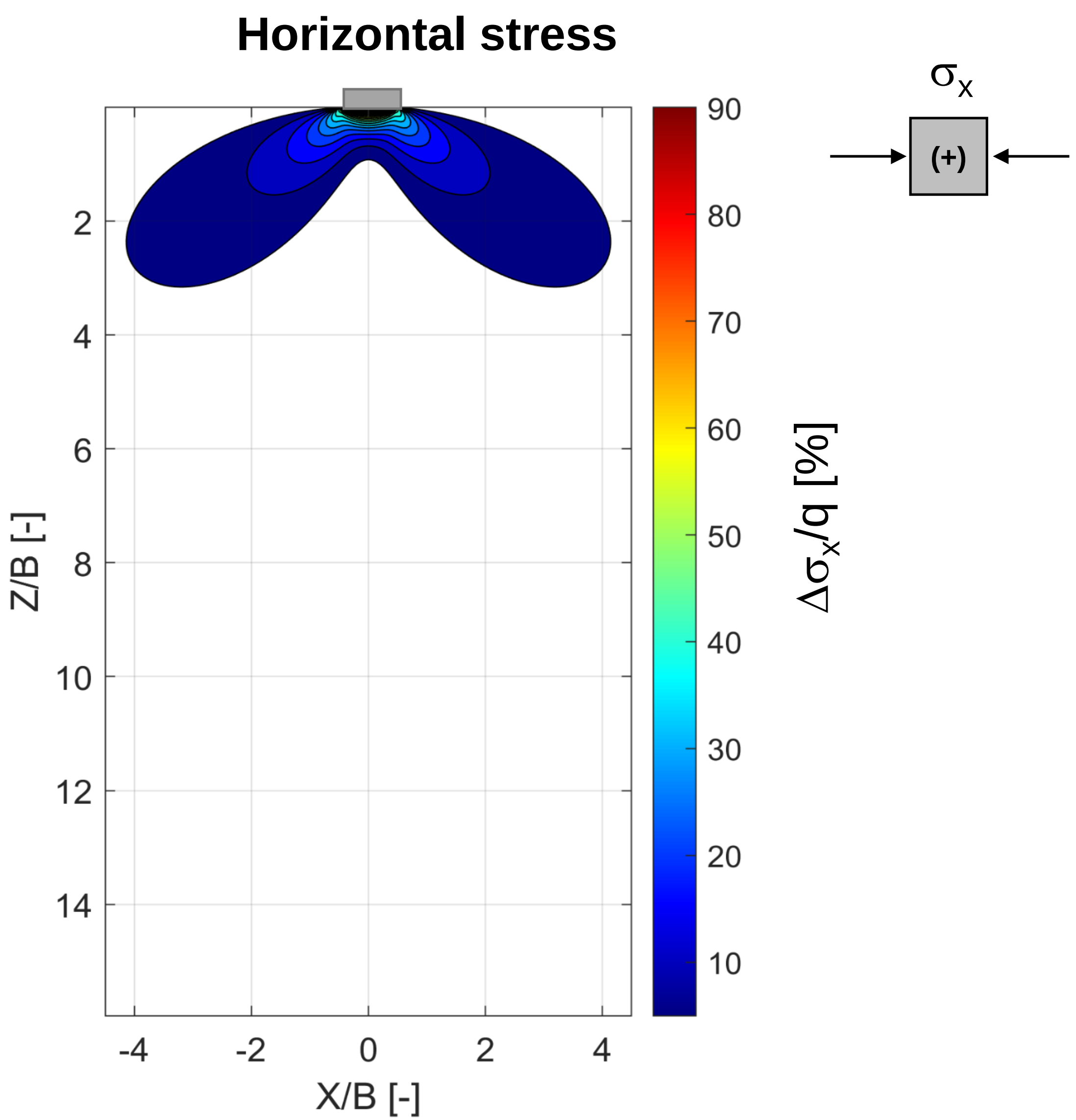
# Contorur figures: vertical stress increment

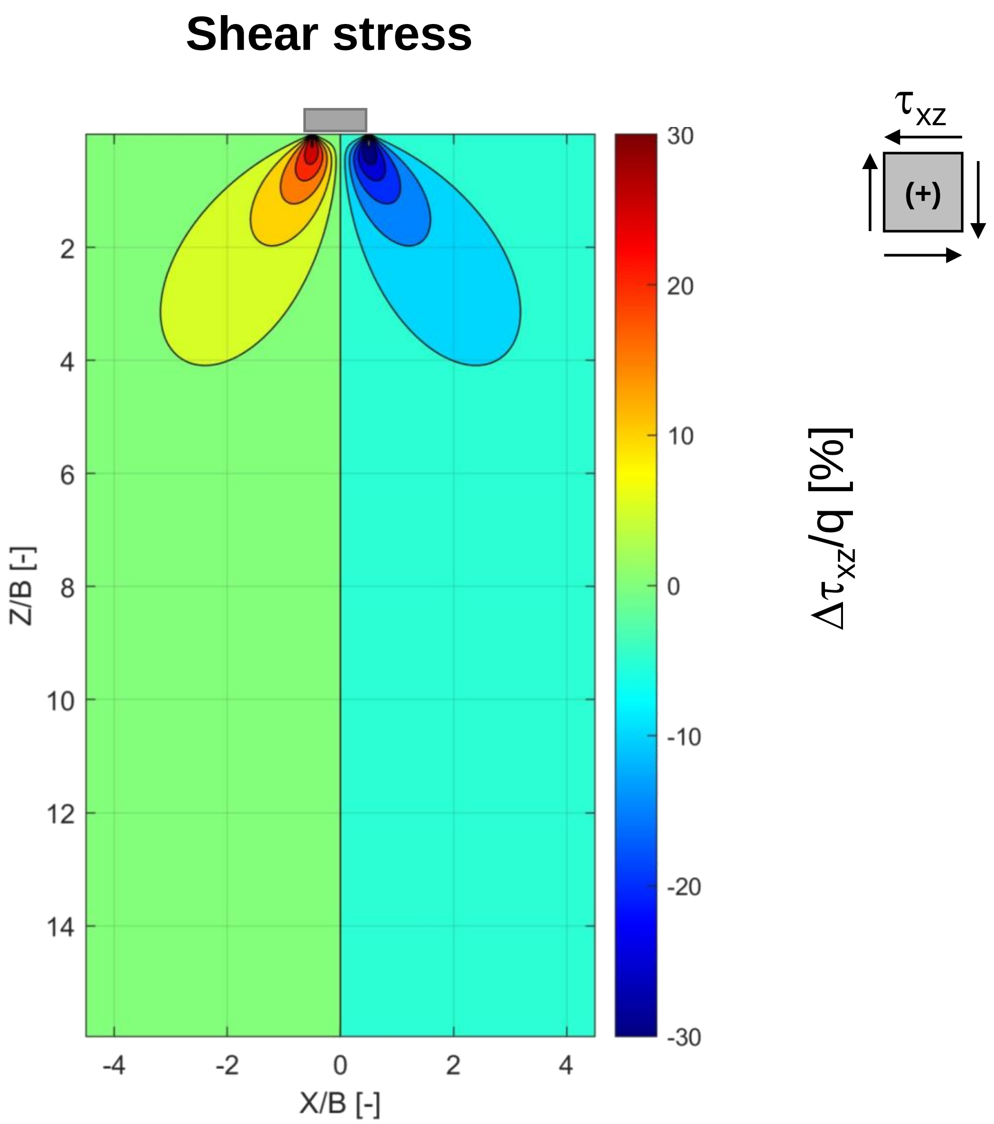


# Contorur figures: horizontal and shear stress increment



# Excel sheets (analytical solution)

Teachers can use it to create additional figures for their classes or provide assignments to students.

| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | **Horizontal coordinated with 0 at the center of loaded area** | | | | | | | | | | | | | | | | | | | |
| 2 | | | **x [m]** | | -9 | -8.9 | -8.8 | -8.7 | -8.6 | -8.5 | -8.4 | -8.3 | -8.2 | -8.1 | -8 | -7.9 | -7.8 | -7.7 | -7.6 | -7.5 | -7.4 | -7.3 | -7.2 | -7 |
| 3 | | | **X/B** | | **-4.5** | **-4.45** | **-4.4** | **-4.35** | **-4.3** | **-4.25** | **-4.2** | **-4.15** | **-4.1** | **-4.05** | **-4** | **-3.95** | **-3.9** | **-3.85** | **-3.8** | **-3.75** | **-3.7** | **-3.65** | **-3.6** | **-3.** |
| 4 | **Vertical coordiante with zero at surface** | **z [m]** | **Z/B** | | **Values of normalized vertical stress increment ($\Delta\sigma_z/q$) at corresponding coordinates** | | | | | | | | | | | | | | | | | | | |
| 5 | | 0.01 | **0.005** | | 2.02E-10 | 2.12E-10 | 2.22E-10 | 2.32E-10 | 2.44E-10 | 2.56E-10 | 2.68E-10 | 2.82E-10 | 2.96E-10 | 3.11E-10 | 3.28E-10 | 3.45E-10 | 3.63E-10 | 3.83E-10 | 4.04E-10 | 4.27E-10 | 4.51E-10 | 4.78E-10 | 5.06E-10 | 5.36E- |
| 6 | | 0.11 | **0.055** | | 2.69E-07 | 2.82E-07 | 2.95E-07 | 3.09E-07 | 3.24E-07 | 3.40E-07 | 3.57E-07 | 3.75E-07 | 3.94E-07 | 4.14E-07 | 4.36E-07 | 4.59E-07 | 4.84E-07 | 5.10E-07 | 5.38E-07 | 5.68E-07 | 6.01E-07 | 6.35E-07 | 6.73E-07 | 7.13E- |
| 7 | | 0.21 | **0.105** | | 1.87E-06 | 1.96E-06 | 2.05E-06 | 2.15E-06 | 2.25E-06 | 2.36E-06 | 2.48E-06 | 2.61E-06 | 2.74E-06 | 2.88E-06 | 3.03E-06 | 3.19E-06 | 3.36E-06 | 3.54E-06 | 3.74E-06 | 3.95E-06 | 4.17E-06 | 4.41E-06 | 4.67E-06 | 4.95E- |
| 8 | | 0.31 | **0.155** | | 6.01E-06 | 6.29E-06 | 6.59E-06 | 6.90E-06 | 7.24E-06 | 7.59E-06 | 7.97E-06 | 8.37E-06 | 8.79E-06 | 9.25E-06 | 9.73E-06 | 1.02E-05 | 1.08E-05 | 1.14E-05 | 1.20E-05 | 1.27E-05 | 1.34E-05 | 1.42E-05 | 1.50E-05 | 1.59E- |
| 9 | | 0.41 | **0.205** | | 1.39E-05 | 1.45E-05 | 1.52E-05 | 1.59E-05 | 1.67E-05 | 1.75E-05 | 1.84E-05 | 1.93E-05 | 2.03E-05 | 2.13E-05 | 2.25E-05 | 2.36E-05 | 2.49E-05 | 2.63E-05 | 2.77E-05 | 2.93E-05 | 3.09E-05 | 3.27E-05 | 3.46E-05 | 3.67E- |
| 10 | | 0.51 | **0.255** | | 2.67E-05 | 2.79E-05 | 2.92E-05 | 3.06E-05 | 3.21E-05 | 3.36E-05 | 3.53E-05 | 3.71E-05 | 3.89E-05 | 4.10E-05 | 4.31E-05 | 4.54E-05 | 4.78E-05 | 5.04E-05 | 5.31E-05 | 5.61E-05 | 5.93E-05 | 6.27E-05 | 6.63E-05 | 7.03E- |
| 11 | | 0.61 | **0.305** | | 4.55E-05 | 4.76E-05 | 4.98E-05 | 5.22E-05 | 5.47E-05 | 5.74E-05 | 6.02E-05 | 6.32E-05 | 6.64E-05 | 6.98E-05 | 7.34E-05 | 7.73E-05 | 8.14E-05 | 8.59E-05 | 9.06E-05 | 9.56E-05 | 0.000101 | 0.000107 | 0.000113 | 0.000 |
| 12 | | 0.71 | **0.355** | | 7.15E-05 | 7.48E-05 | 7.83E-05 | 8.20E-05 | 8.59E-05 | 9.01E-05 | 9.46E-05 | 9.93E-05 | 0.000104 | 0.00011 | 0.000115 | 0.000121 | 0.000128 | 0.000135 | 0.000142 | 0.00015 | 0.000158 | 0.000168 | 0.000177 | 0.0001 |
| 13 | | 0.81 | **0.405** | | 0.000106 | 0.000111 | 0.000116 | 0.000121 | 0.000127 | 0.000133 | 0.00014 | 0.000147 | 0.000154 | 0.000162 | 0.00017 | 0.000179 | 0.000189 | 0.000199 | 0.00021 | 0.000221 | 0.000234 | 0.000247 | 0.000262 | 0.0002 |
| 14 | | 0.91 | **0.455** | | 0.000149 | 0.000156 | 0.000163 | 0.000171 | 0.000179 | 0.000188 | 0.000197 | 0.000207 | 0.000217 | 0.000228 | 0.00024 | 0.000253 | 0.000266 | 0.00028 | 0.000296 | 0.000312 | 0.000329 | 0.000348 | 0.000368 | 0.000 |
| 15 | | 1.01 | **0.505** | | 0.000203 | 0.000212 | 0.000222 | 0.000233 | 0.000244 | 0.000256 | 0.000268 | 0.000281 | 0.000295 | 0.00031 | 0.000326 | 0.000343 | 0.000362 | 0.000381 | 0.000402 | 0.000424 | 0.000447 | 0.000472 | 0.0005 | 0.0005 |
| 16 | | 1.11 | **0.555** | | 0.000268 | 0.00028 | 0.000293 | 0.000307 | 0.000322 | 0.000337 | 0.000354 | 0.000371 | 0.00039 | 0.000409 | 0.00043 | 0.000453 | 0.000476 | 0.000502 | 0.000529 | 0.000558 | 0.000589 | 0.000622 | 0.000657 | 0.0006 |
| 17 | | 1.21 | **0.605** | | 0.000345 | 0.000361 | 0.000378 | 0.000395 | 0.000414 | 0.000434 | 0.000455 | 0.000477 | 0.000501 | 0.000526 | 0.000553 | 0.000582 | 0.000612 | 0.000645 | 0.000679 | 0.000716 | 0.000756 | 0.000798 | 0.000844 | 0.0008 |
| 18 | | 1.31 | **0.655** | | 0.000435 | 0.000455 | 0.000476 | 0.000498 | 0.000522 | 0.000547 | 0.000573 | 0.000601 | 0.000631 | 0.000663 | 0.000696 | 0.000732 | 0.00077 | 0.000811 | 0.000854 | 0.000901 | 0.00095 | 0.001003 | 0.00106 | 0.0011 |
| 19 | | 1.41 | **0.705** | | 0.000539 | 0.000563 | 0.000589 | 0.000617 | 0.000646 | 0.000677 | 0.000709 | 0.000744 | 0.00078 | 0.000819 | 0.000861 | 0.000905 | 0.000952 | 0.001002 | 0.001055 | 0.001112 | 0.001173 | 0.001238 | 0.001307 | 0.0013 |
| 20 | | 1.51 | **0.755** | | 0.000657 | 0.000687 | 0.000718 | 0.000752 | 0.000787 | 0.000824 | 0.000864 | 0.000906 | 0.00095 | 0.000997 | 0.001047 | 0.001101 | 0.001158 | 0.001218 | 0.001283 | 0.001351 | 0.001425 | 0.001503 | 0.001587 | 0.0016 |
| 21 | | 1.61 | **0.805** | | 0.00079 | 0.000826 | 0.000864 | 0.000903 | 0.000946 | 0.00099 | 0.001037 | 0.001088 | 0.001141 | 0.001197 | 0.001257 | 0.001321 | 0.001389 | 0.001461 | 0.001538 | 0.00162 | 0.001707 | 0.001801 | 0.001901 | 0.0020 |
| 22 | | 1.71 | **0.855** | | 0.000939 | 0.000981 | 0.001026 | 0.001073 | 0.001123 | 0.001175 | 0.001231 | 0.00129 | 0.001353 | 0.00142 | 0.001491 | 0.001566 | 0.001646 | 0.001731 | 0.001821 | 0.001918 | 0.002021 | 0.002131 | 0.002248 | 0.0023 |
| 23 | | 1.81 | **0.905** | | 0.001104 | 0.001153 | 0.001205 | 0.001261 | 0.001319 | 0.00138 | 0.001446 | 0.001515 | 0.001588 | 0.001666 | 0.001748 | 0.001836 | 0.001929 | 0.002028 | 0.002133 | 0.002246 | 0.002366 | 0.002493 | 0.00263 | 0.0027 |
| 24 | | 1.91 | **0.955** | | 0.001285 | 0.001343 | 0.001403 | 0.001467 | 0.001534 | 0.001605 | 0.001681 | 0.001761 | 0.001845 | 0.001935 | 0.002031 | 0.002132 | 0.002239 | 0.002353 | 0.002475 | 0.002604 | 0.002742 | 0.002889 | 0.003046 | 0.0032 |

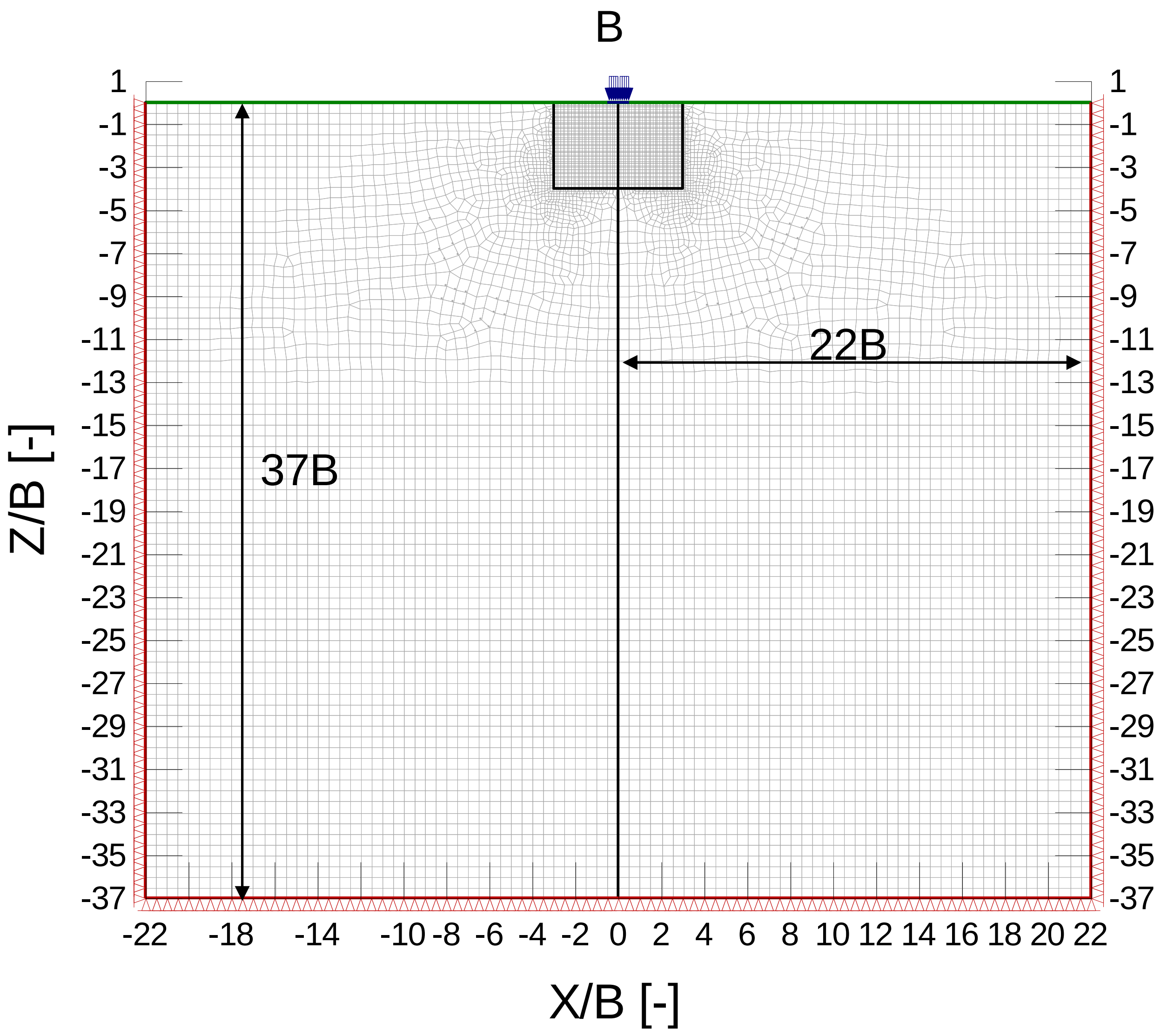
Dimensions and load | DeltaSigmaZ | DeltaSigmaX | DeltaTau | DeltaSigmaZ_norm | DeltaSigmaX_norm | DeltaTa ...

# Numerical solution

GeoStudio 2024

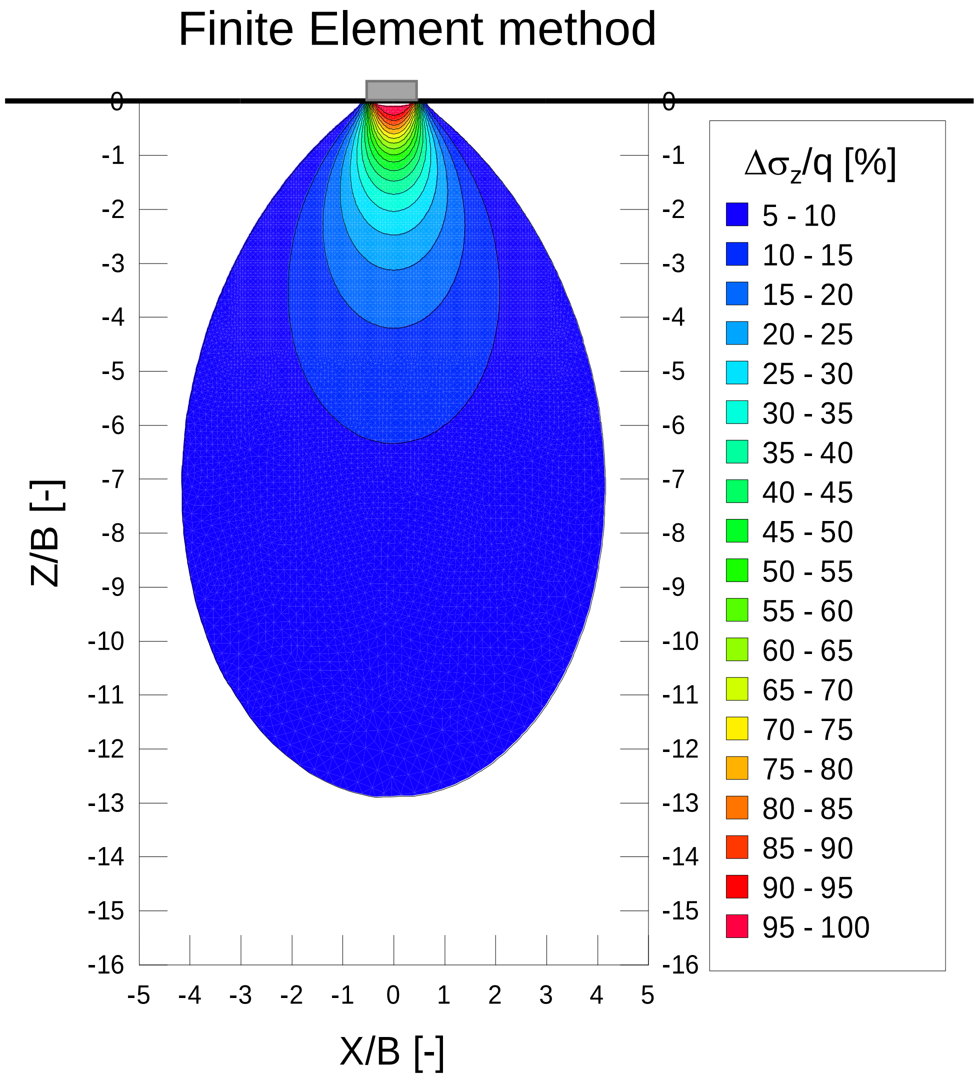
| Feature | Model 1 (Analytical Comparison) | Model 2 (Elastic - Medium Dense Sand) | Model 3 (Elastic - Soft Clay) | Model 4 (Mohr-Coulomb - Medium Dense Sand) | Model 5 (Modified Cam Clay) | Model 6 (Modified Cam Clay - Excavation) |
|---|---|---|---|---|---|---|
| Load Entity | 100kPa | | | | | |
| Load Width | 1m | | | | | |
| Material Model | Linear Elastic | | | | | |
| Material parameters | E=30000kPa, $\nu$=0.3 | | | | | |
| Soil Unit Weight | 0 | | | | | |
| Notes | **No gravity, used for comparison with analytical solution** | | | | | |

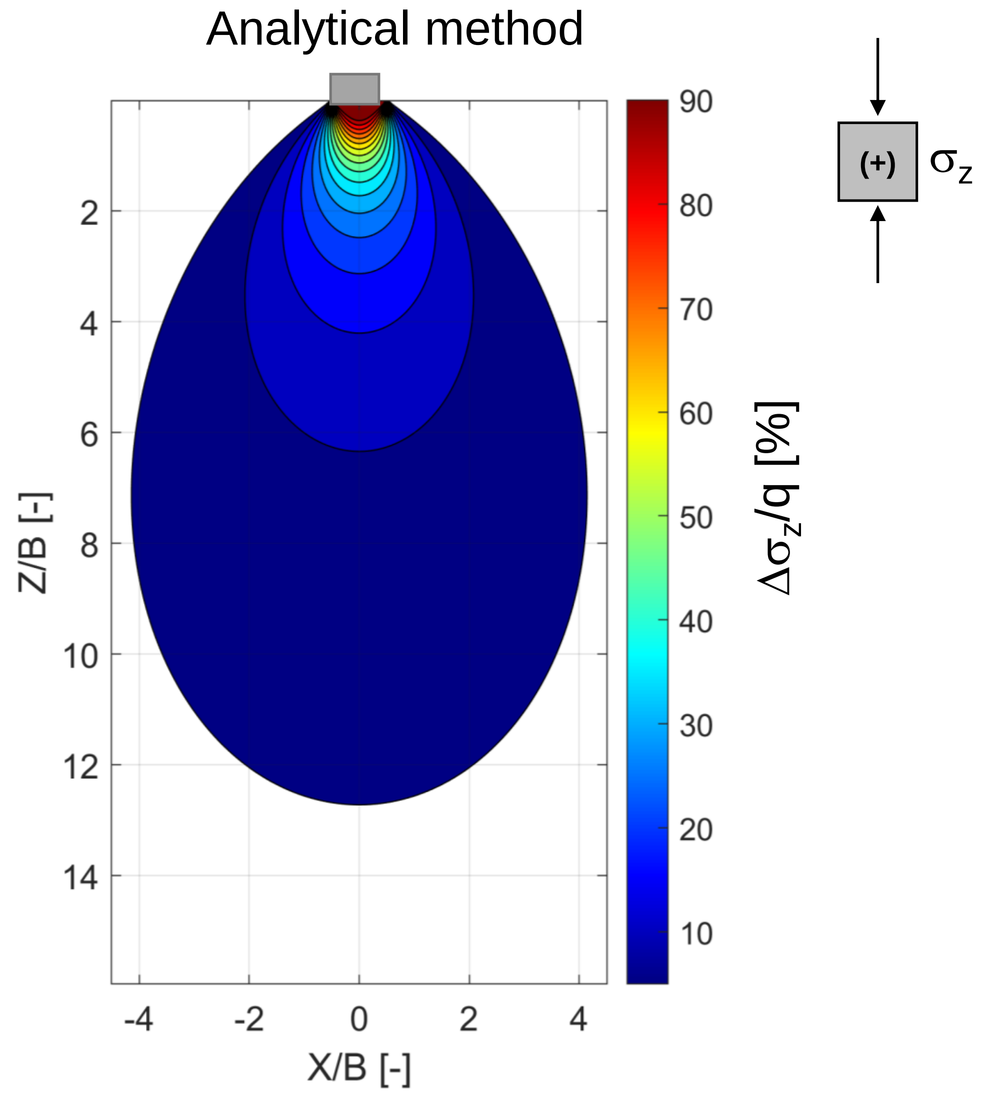
# Model 1: geometry and boundary conditions




*Will you show it to your students?*

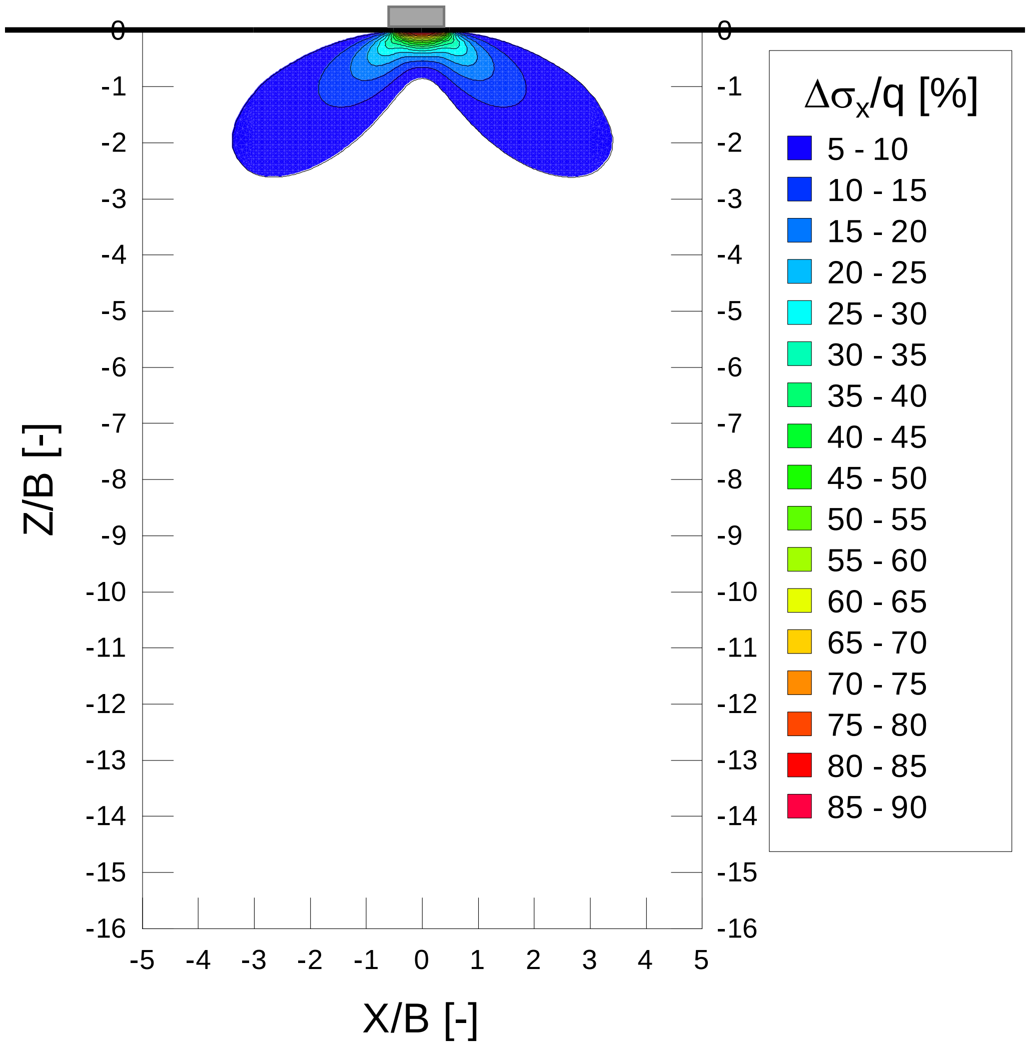
# Increment of vertical stress due to a uniform, flexible strip load



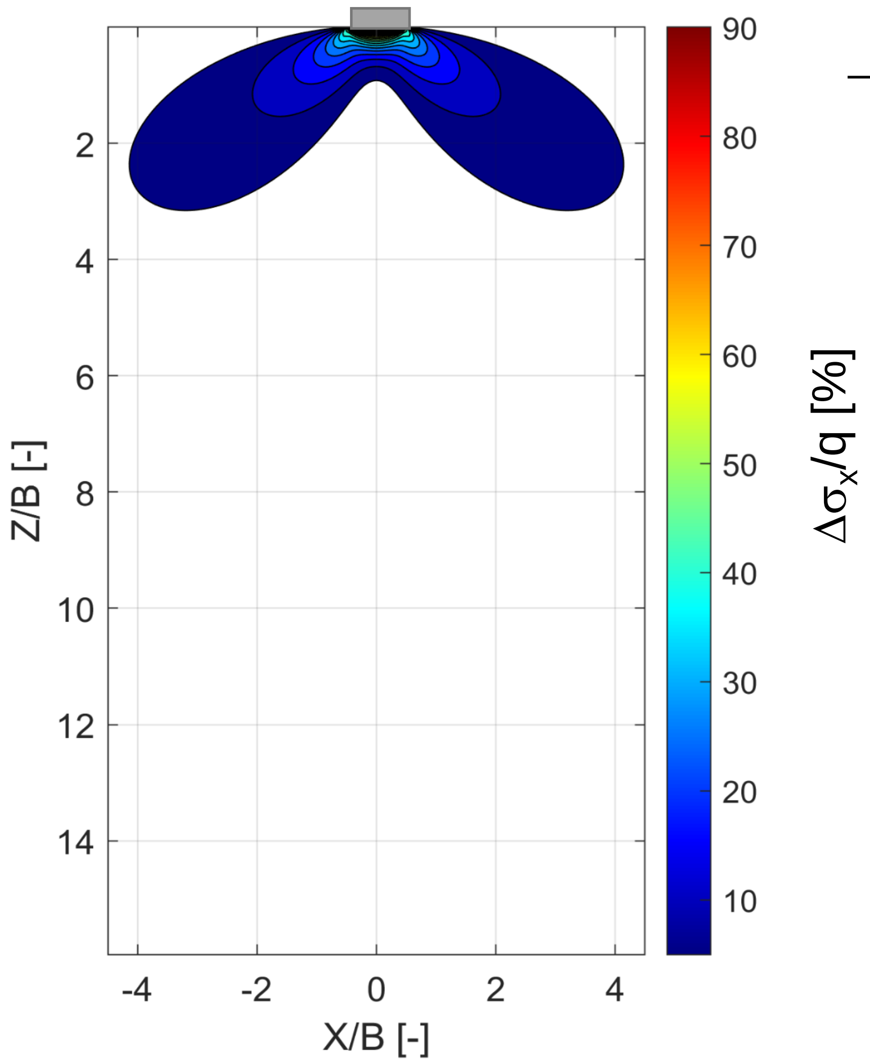



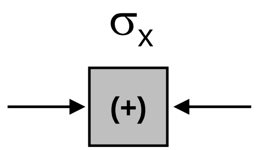
# Increment of horizontal stress due to a uniform strip load

Finite Element method



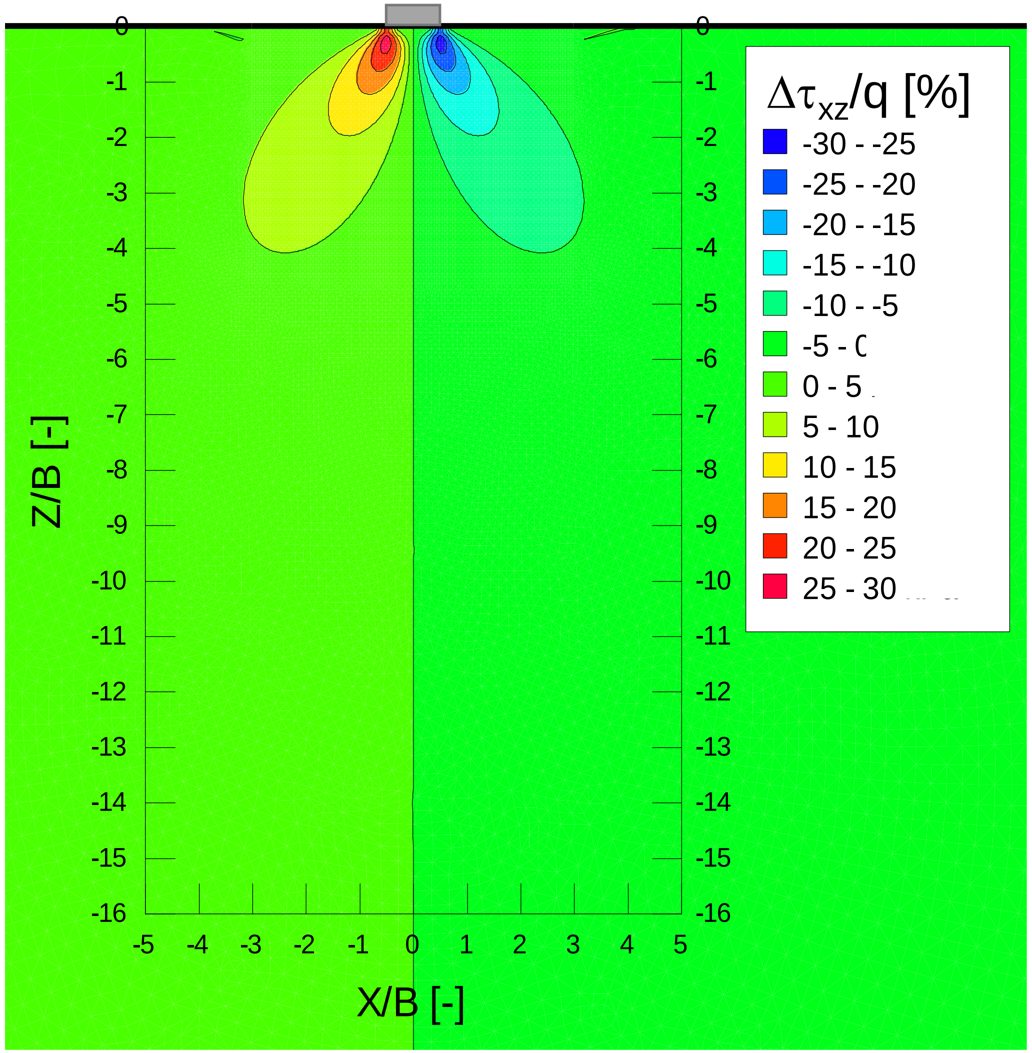

Analytical method



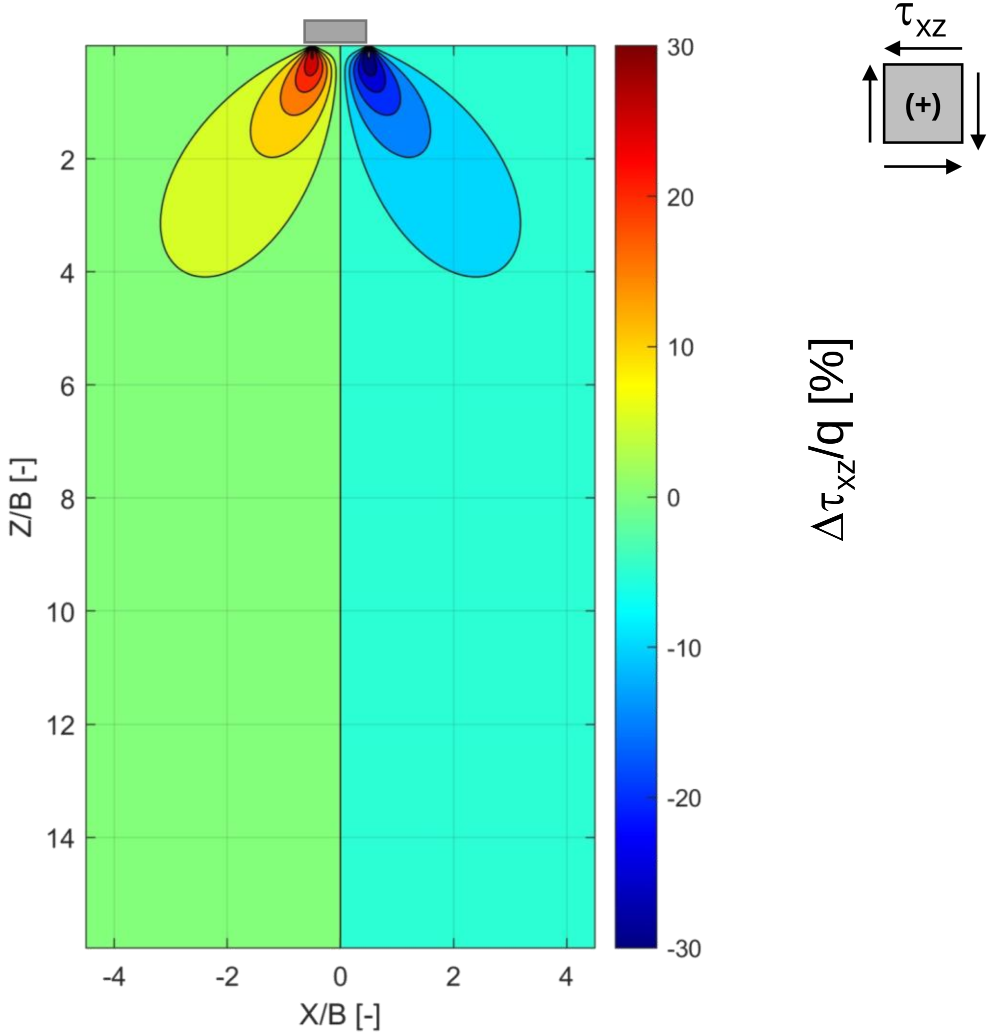



# Increment of shear stress due to a uniform strip load

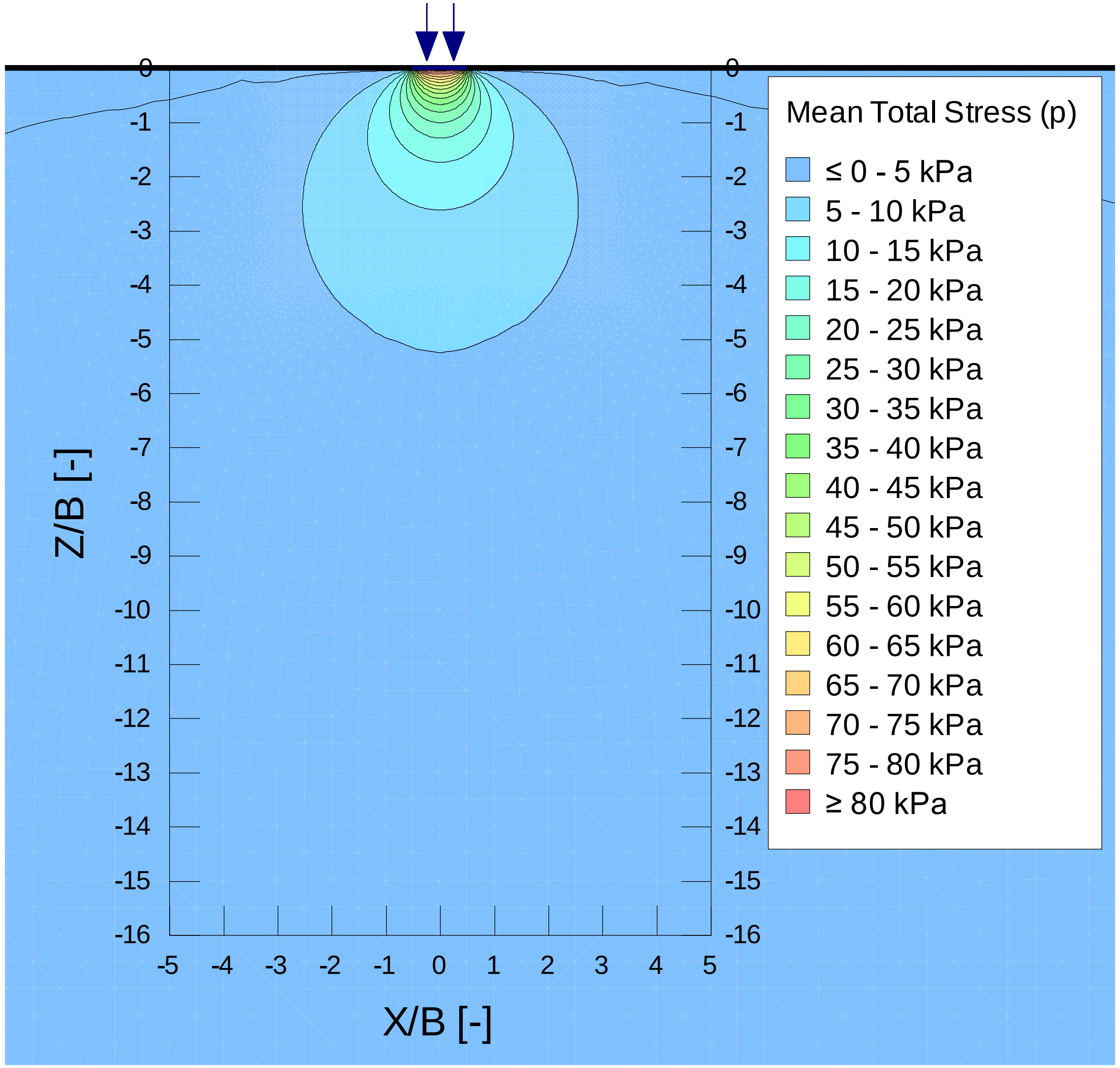
Finite Element method

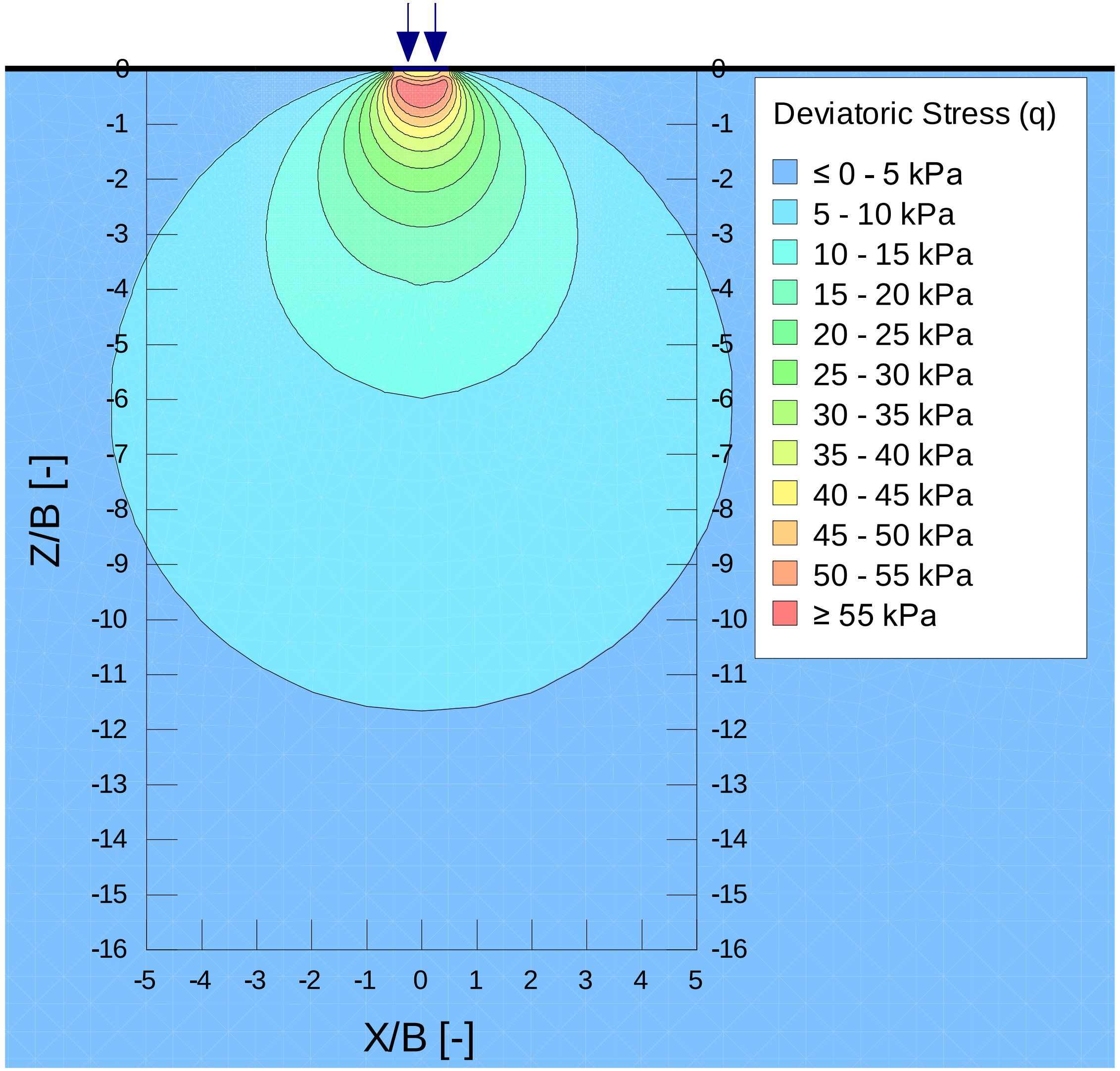



Analytical method



# Mean stress and deviatoric stress increment due to a uniform strip load

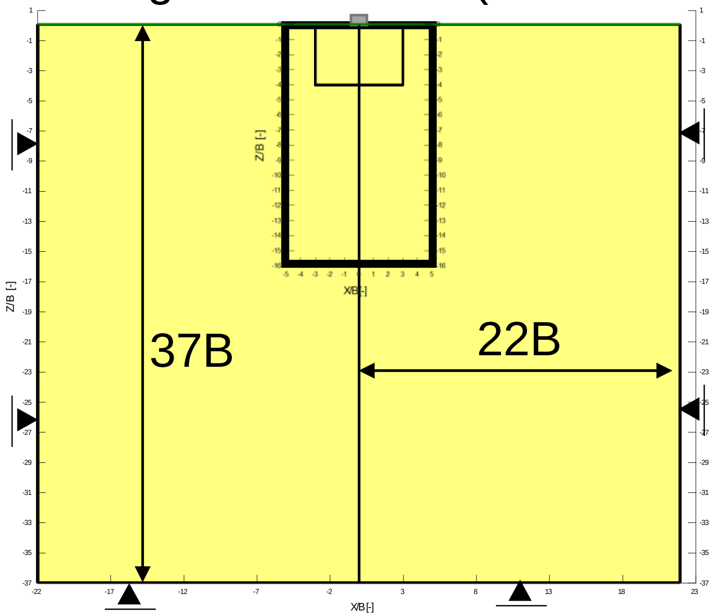


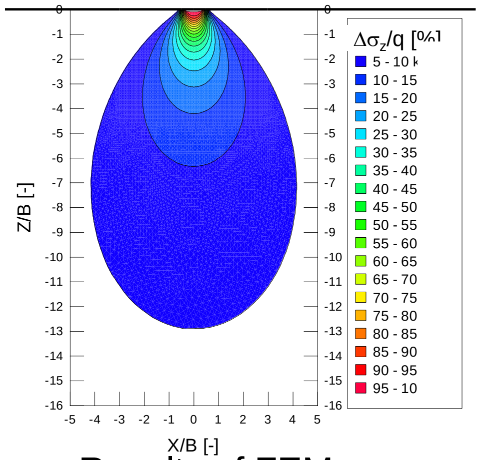
# Increment of vertical stress due to a uniform strip load:

## Effect of model size in FEM

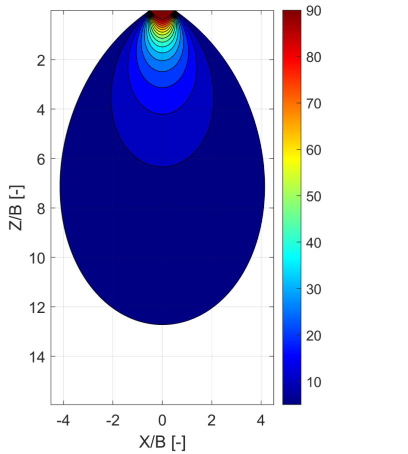
Large FEM model (far boundaries)




Good agreement with analytical solution

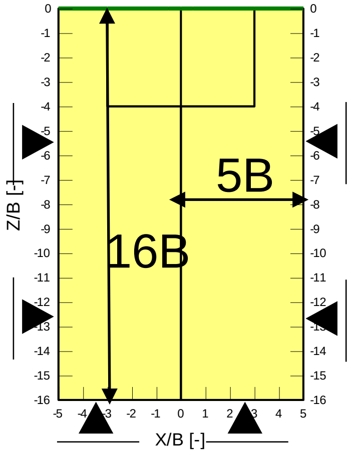



Results of FEM

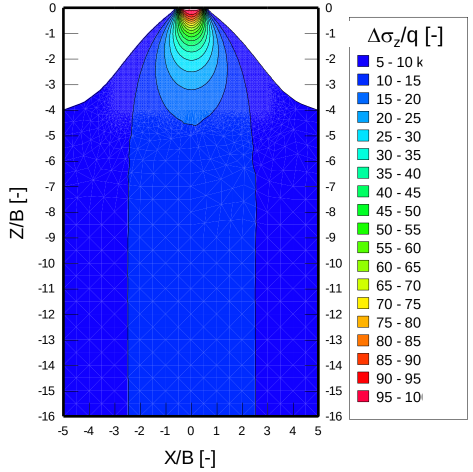



Results analytical solution

Small FEM model (close boundaries)




Results of FEM




Bad agreement with analytical solution!

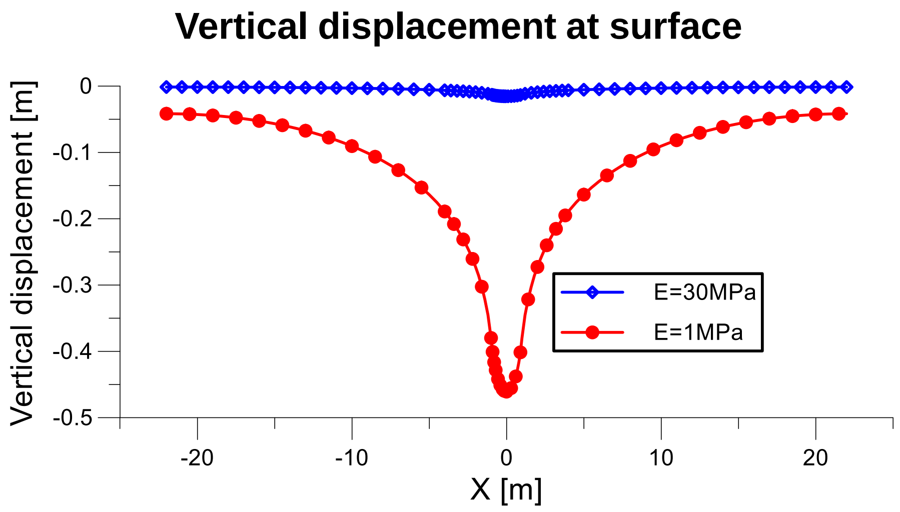
# Numerical solution

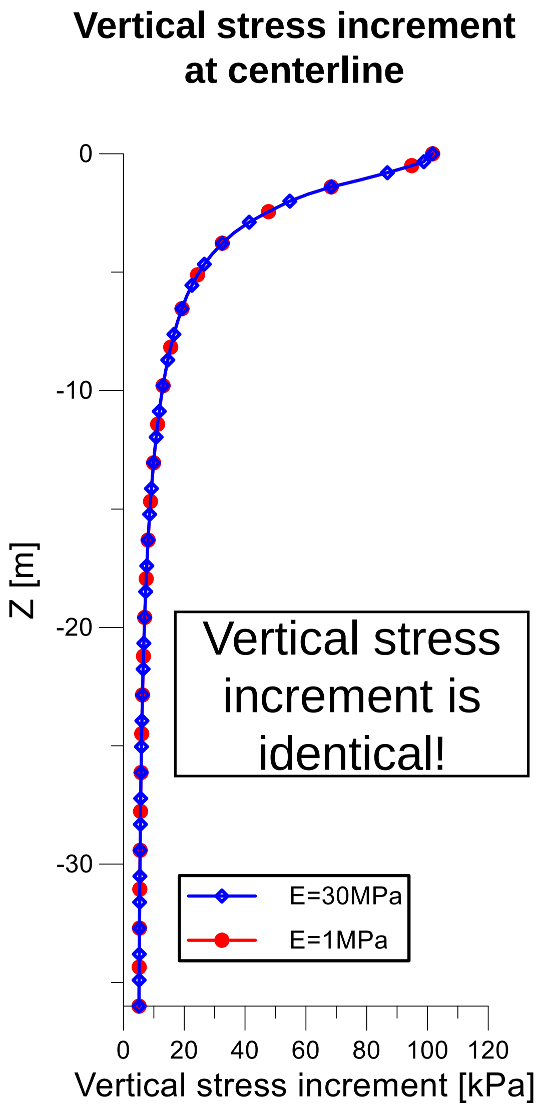
GeoStudio 2024

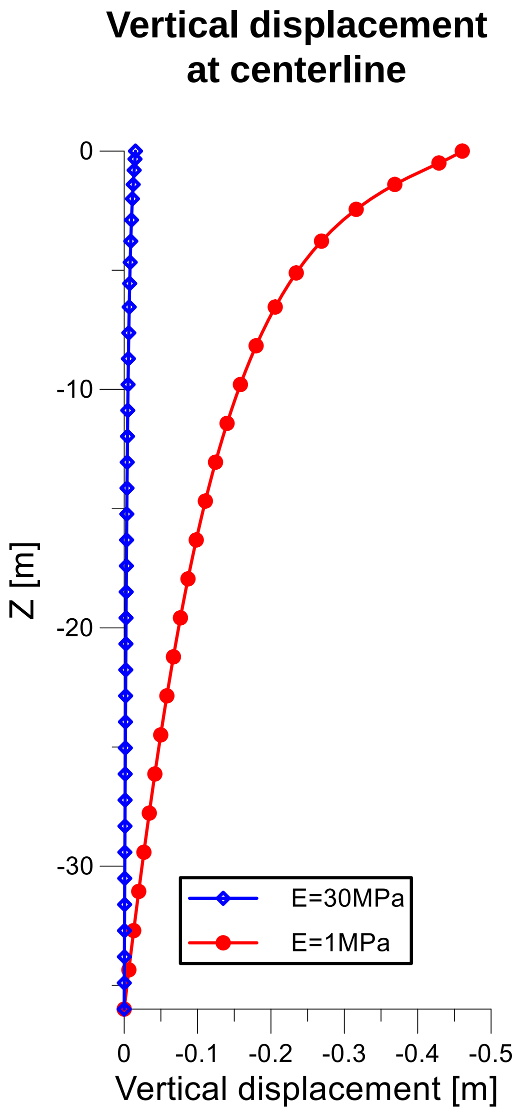
| Feature | Model 1 (Analytical Comparison) | Model 2 (Elastic - Medium Dense Sand) | Model 3 (Elastic - Soft Clay) | Model 4 (Mohr-Coulomb - Medium Dense Sand) | Model 5 (Modified Cam Clay) | Model 6 (Modified Cam Clay - Excavation) |
|---|---|---|---|---|---|---|
| Load Entity | 100kPa | 100kPa | 100kPa | | | |
| Load Width | 1m | 2m | 2m | | | |
| Material Model | Linear Elastic | Linear Elastic | Linear Elastic | | | |
| Material parameters | E=30000kPa, $\nu$=0.3 | **E=30000kPa** $\nu$=0.3 | **E=1000kPa** $\nu$=0.3 | | | |
| Soil Unit Weight | 0 | 20 kN/m³ | 20 kN/m³ | | | |
| Notes | **No gravity, used for comparison with analytical solution** | Simulates medium dense sand | Simulates soft clay | | | |

# Vertical displacement due to a uniform strip load: effect of Young modulus

**Learning outcome**: displacement are larger in soft soils even if the stress increment is identical.



# Numerical solution

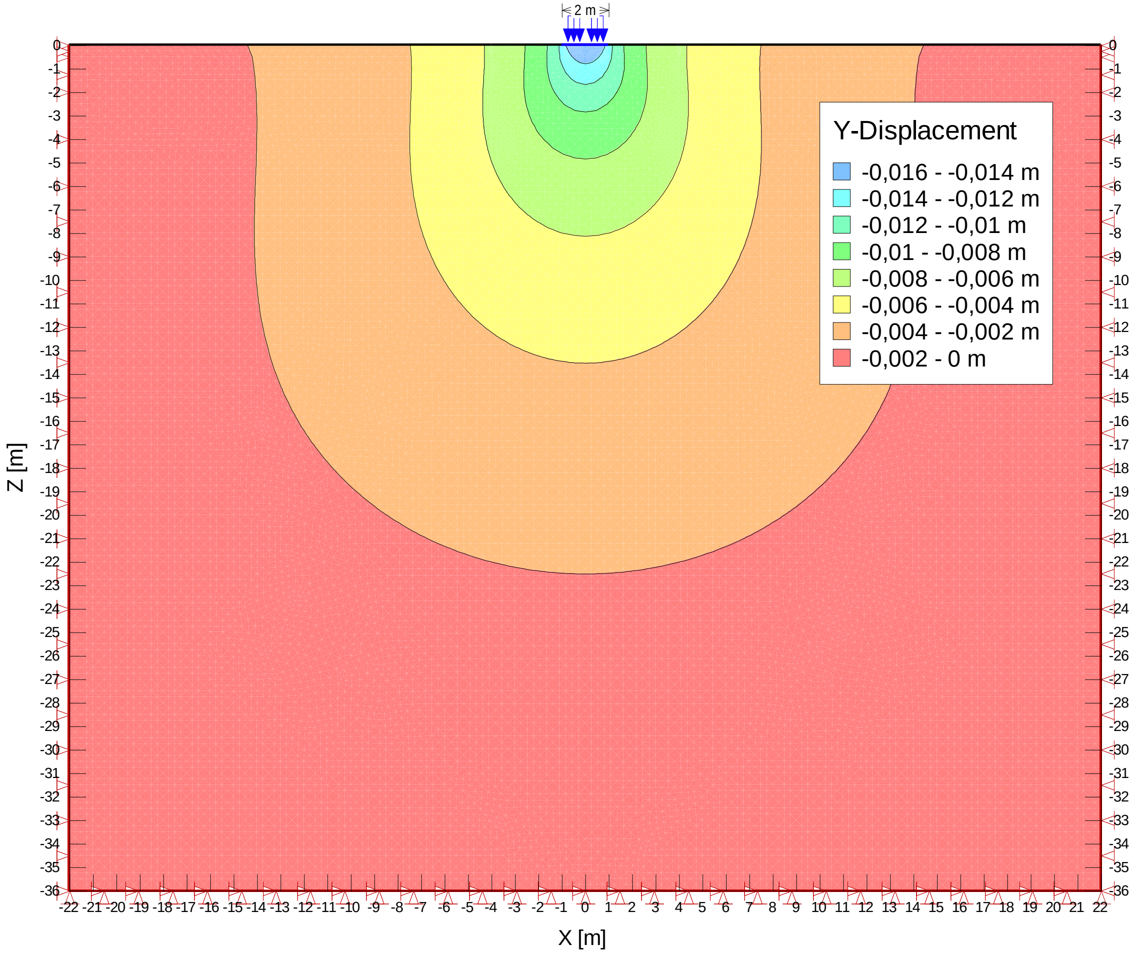
GeoStudio 2024

| Feature | Model 1 (Analytical Comparison) | Model 2 (Elastic - Medium Dense Sand) | Model 3 (Elastic - Soft Clay) | Model 4 (Mohr-Coulomb - Medium Dense Sand) | Model 5 (Modified Cam Clay) | Model 6 (Modified Cam Clay - Excavation) |
| --- | --- | --- | --- | --- | --- | --- |
| **Load Entity** | 100kPa | 100kPa | 100kPa | 100kPa | | |
| **Load Width** | 1m | 2m | 2m | 2m | | |
| **Material Model** | Linear Elastic | Linear Elastic | Linear Elastic | **Elastic-Perfectly Plastic (Mohr-Coulomb)** | | |
| **Material parameters** | E=30000kPa, $\nu$=0.3 | **E=30000kPa** $\nu$=0.3 | **E=1000kPa** $\nu$=0.3 | E=30000kPa $\nu$=0.3 $\varphi$=35° | | |
| **Soil Unit Weight** | 0 | 20 kN/m³ | 20 kN/m³ | 20 kN/m³ | | |
| **Notes** | **No gravity, used for comparison with analytical solution** | Simulates medium dense sand | Simulates soft clay | Sher failure mechanism included in the model | | |

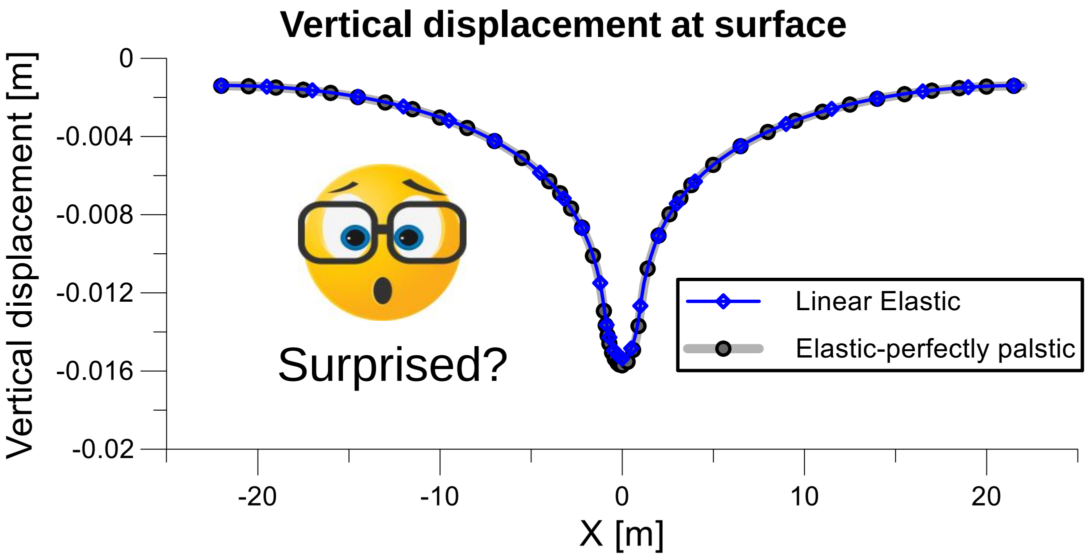
# Vertical displacment due to a uniform strip load:

## Linear elastic (LE) vs elastic-perfectly plastic soil (EP) with Mohr-Coulomb failure criterion

Contour of vertical displacement in FEM analysis: LE and EP are identical!




**Learning outcome**: when the load is small, the displacements calculate with LE and EP is identical.



# Numerical solution

GeoStudio 2024

| Feature | Model 1 (Analytical Comparison) | Model 2 (Elastic - Medium Dense Sand) | Model 3 (Elastic - Soft Clay) | Model 4 (Mohr-Coulomb - Medium Dense Sand) | Model 5 (Modified Cam Clay) | Model 6 (Modified Cam Clay - Excavation) |
|---|---|---|---|---|---|---|
| **Load Entity** | 100kPa | 100kPa | 100kPa | 100kPa | 100kPa | 100kPa |
| **Load Width** | 1m | 2m | 2m | 2m | 2m | 2m (applied at excavation bottom) |
| **Material Model** | Linear Elastic | Linear Elastic | Linear Elastic | **Elastic-Perfectly Plastic (Mohr-Coulomb)** | **Modified Cam Clay** | **Modified Cam Clay** |
| **Material parameters** | E=30000kPa, ν=0.3 | **E=30000kPa** ν=0.3 | **E=1000kPa** ν=0.3 | E=30000kPa ν=0.3 φ=35° | Table 2 | Table 2 |
| **Soil Unit Weight** | 0 | 20 kN/m³ | 20 kN/m³ | 20 kN/m³ | 20 kN/m³ | 20 kN/m³ |
| **Notes** | **No gravity, used for comparison with analytical solution** | Simulates medium dense sand | Simulates soft clay | Sher failure mechanism included in the model | This model has **convergence issues** | Load applied at excavation bottom |

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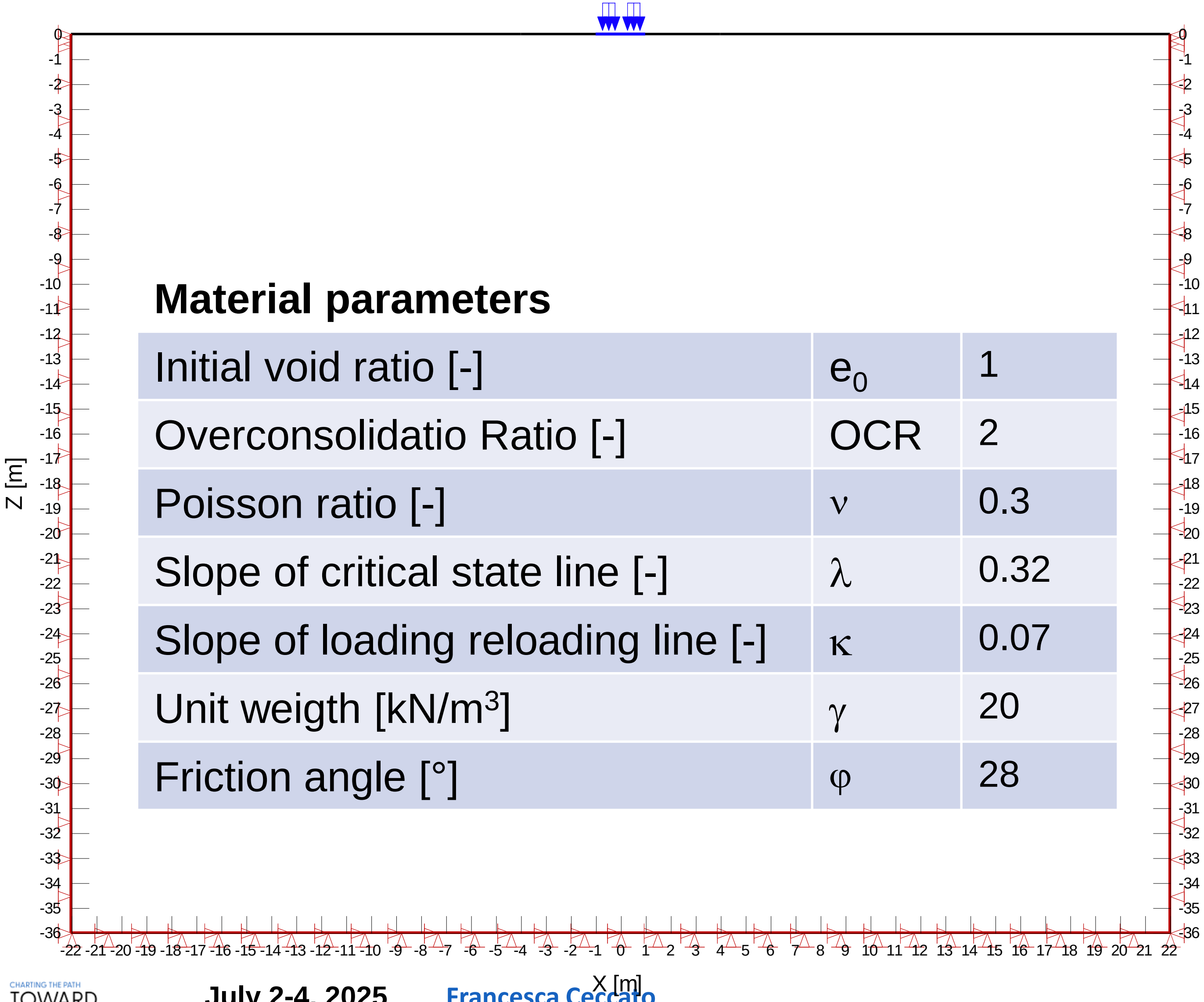
# Vertical displacement due to a uniform strip load on clay:

## Modified Cam Clay Model (MCC) vs Linear Elastic model (LE)– load on the surface




**Material parameters**

| | | |
|---|---|---|
| Initial void ratio [-] | $e_0$ | 1 |
| Overconsolidatio Ratio [-] | OCR | 2 |
| Poisson ratio [-] | $\nu$ | 0.3 |
| Slope of critical state line [-] | $\lambda$ | 0.32 |
| Slope of loading reloading line [-] | $\kappa$ | 0.07 |
| Unit weigth [kN/m$^3$] | $\gamma$ | 20 |
| Friction angle [°] | $\varphi$ | 28 |

Load q=100kPa applied on the surface.

The simulation does not converge!

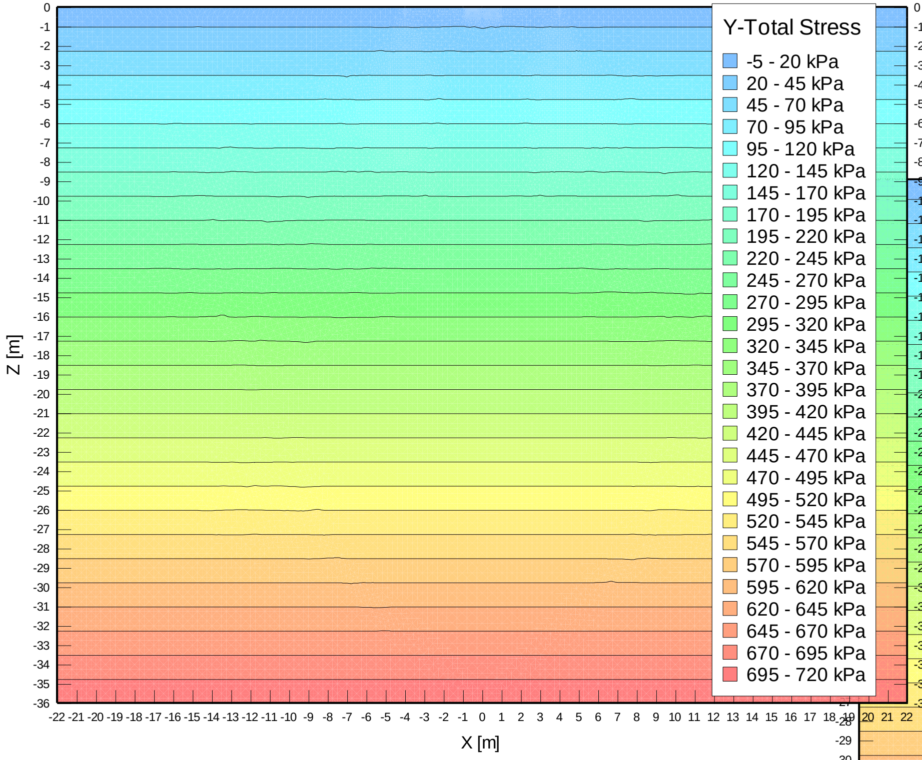
*How will you explain it to your students?*

# Vertical displacement due to a uniform strip load on clay:

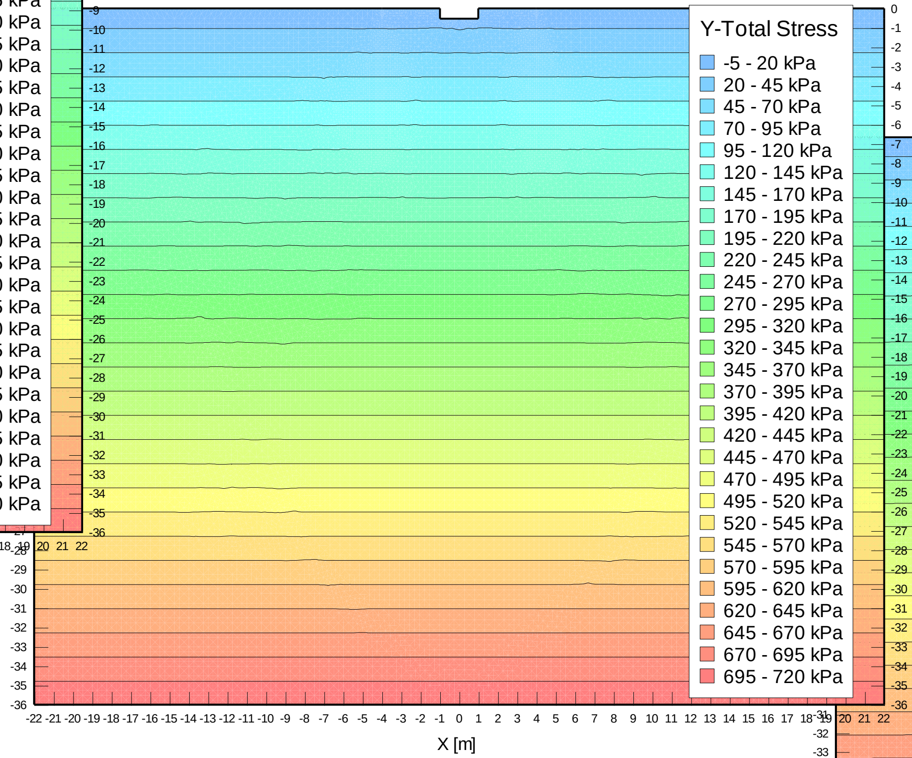
## Modified Cam Clay Model (MCC) vs Linear Elastic model (LE)– load on a small excavation

Calculation steps:

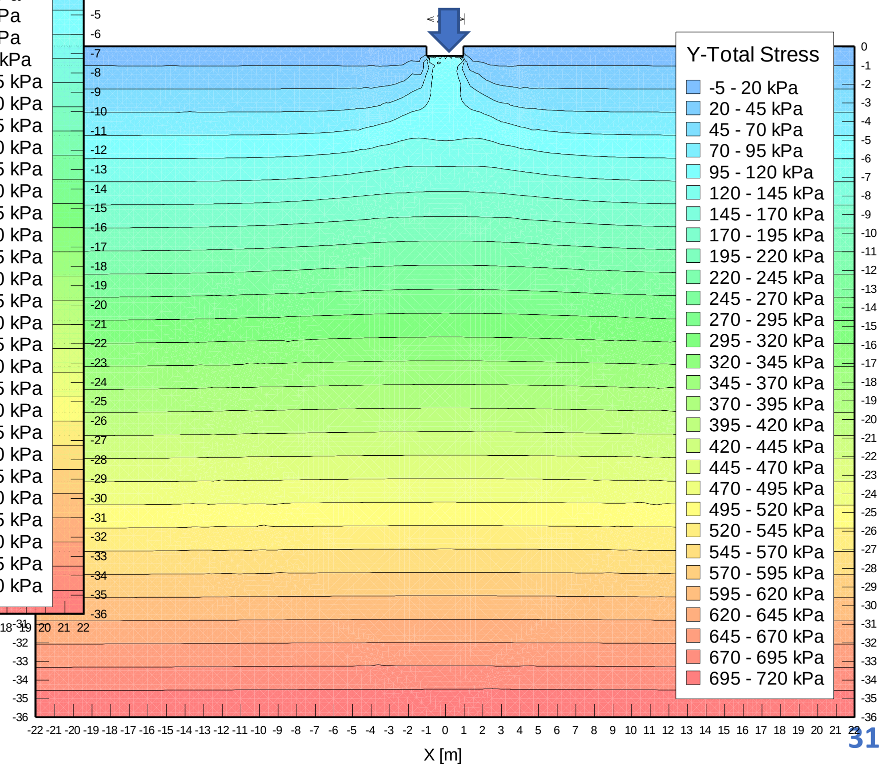
1. Generation of in-situ stresses




2. Small excavation (0.5m)



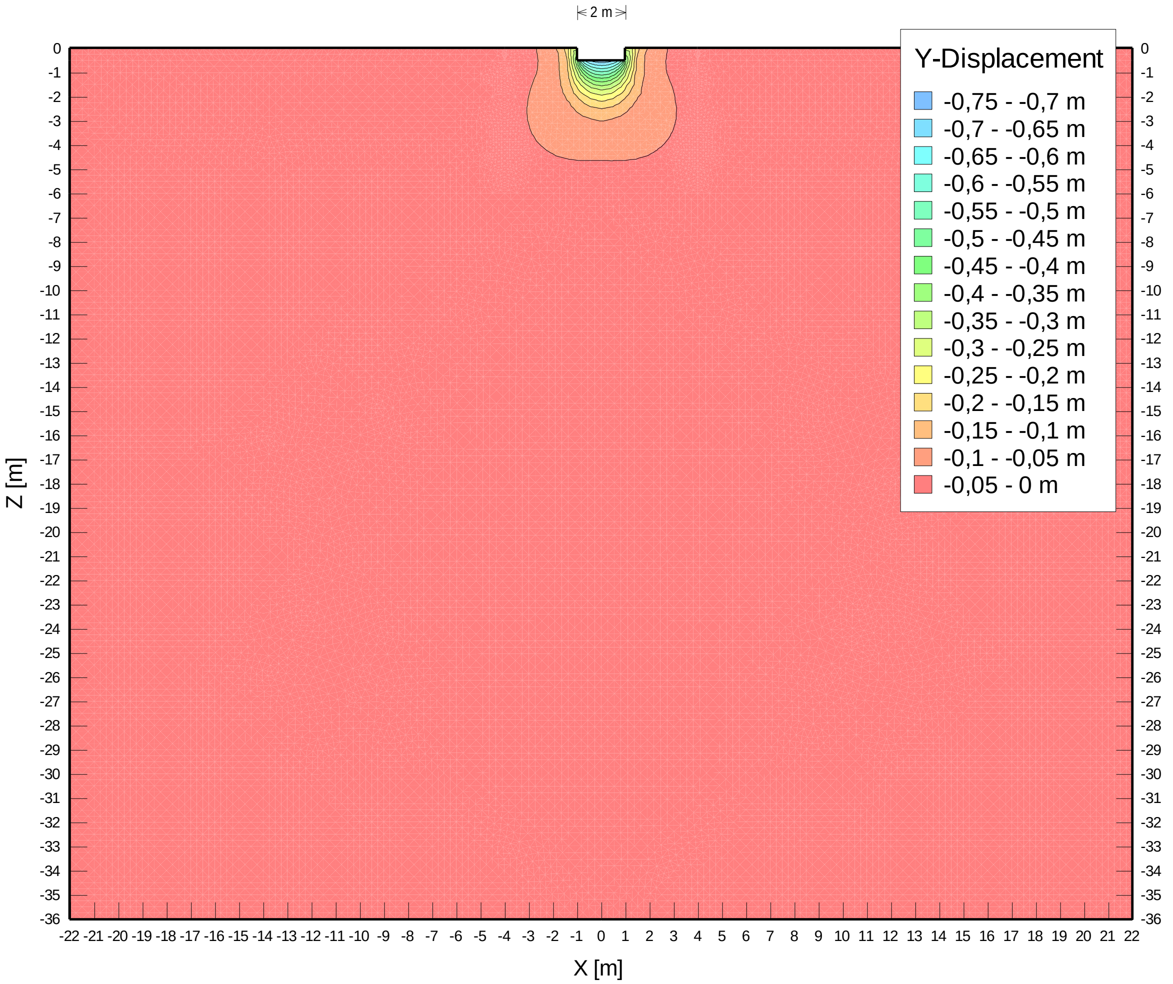

3. Load application



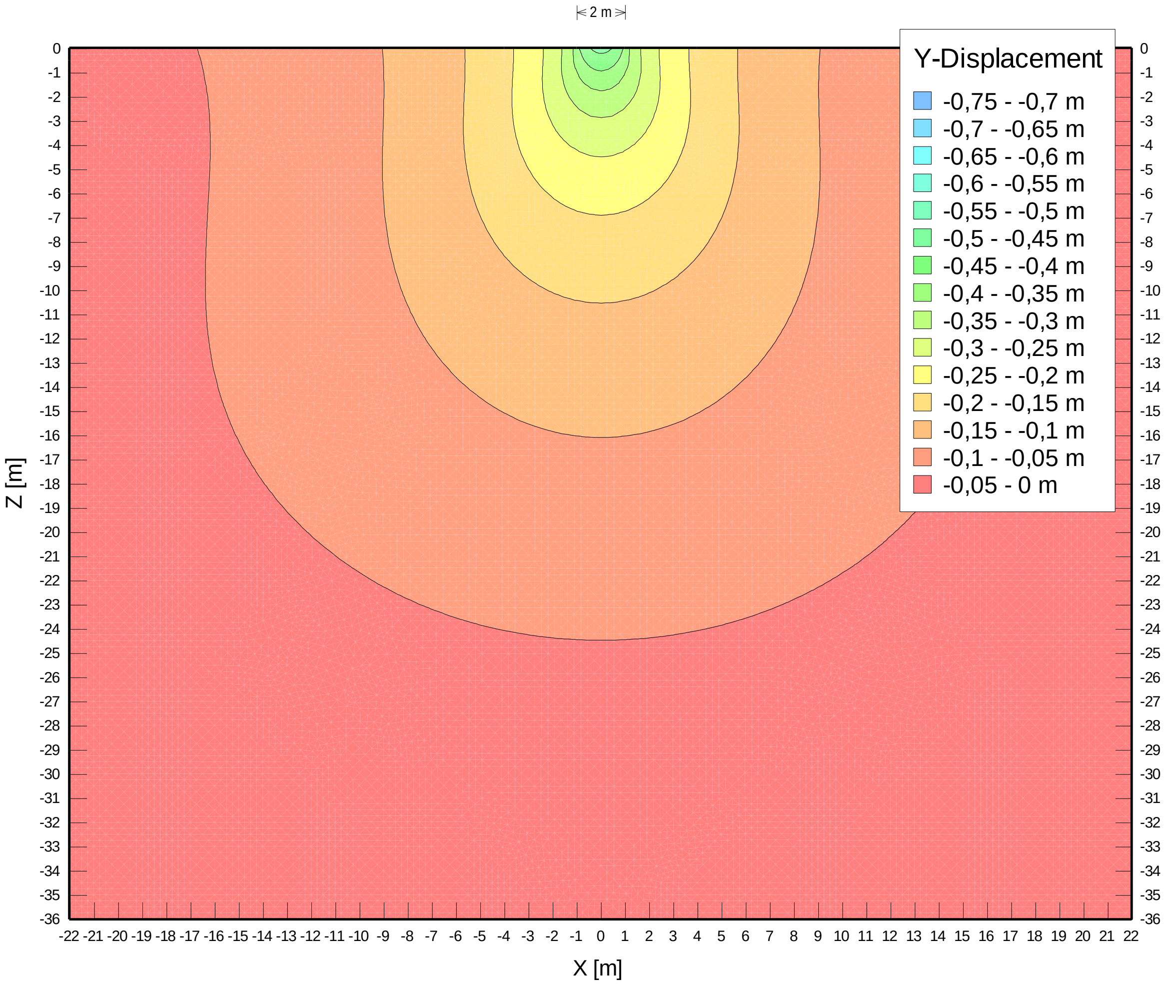
# Vertical displacement due to a uniform strip load on clay:

## Modified Cam Clay Model (MCC) vs Linear Elastic model (LE)– load on a small excavation

### Modified Cam Clay Model



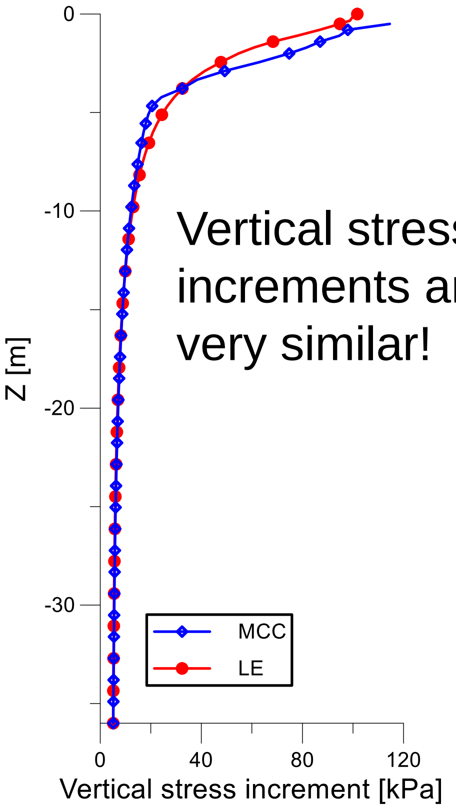

### Linear Elastic model (E=1000kPa, $\nu$=0.3)



# Vertical displacement due to a uniform strip load on clay:

## Modified Cam Clay Model (MCC) vs Linear Elastic model (LE)– load on a small excavation

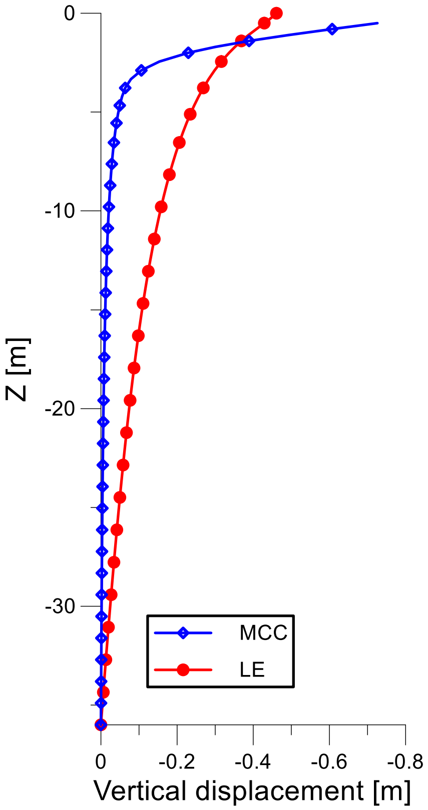
**Vertical stress increment at centerline**



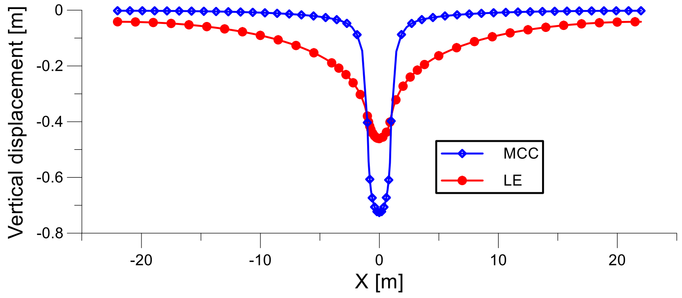

Vertical stress increments are very similar!

Vertical displacements are totally different!

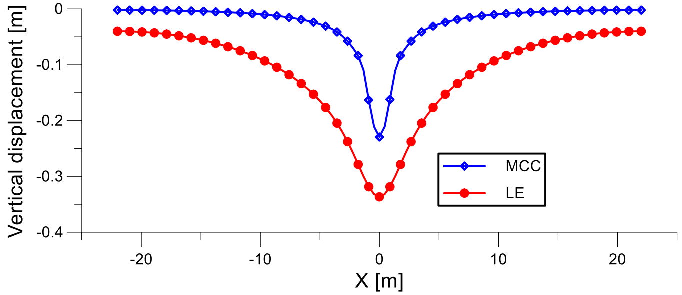
**Vertical displacement at centerline**



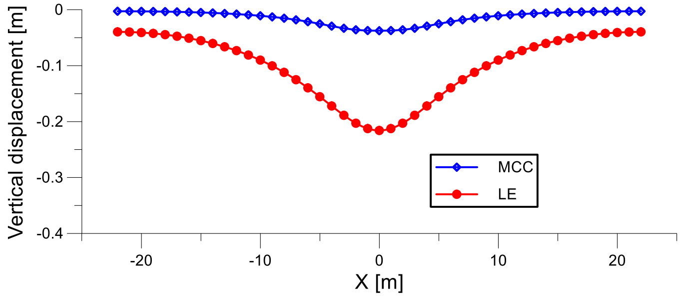

**Vertical displacement at depth of foundation plane**




**Vertical displacement at Z=-2m**




**Vertical displacement at Z=-6m**



# Using Material in Basic Courses

- Enhance student learning experience
- Improve digital litteracy
- Improve fundamental understanding of soil behavior
- Visualize stress distributions, displacements
- Improve critical thinking

# Using Materials in Advanced Courses

- Enhance critical thinking
- Examples for discussion
- Model limitations
- Assignments involving model implementation, variations

# Effort Required & Importance Collaboration

- Preparing high-quality educational materials requires a significant effort:
  - Coding (1.5h Matlab),
  - model setup (2h FEM),
  - post-processing (10h),
  - Documentation (5h).
- Review of educational material requires significant effort

An engaged Geotechnical Engineering Educator community is necessary!

# Conclusions

- Collaborative initiative for high-quality educational materials
- Preparing high-quality materials requires significant effort (coding, modeling, visualization, documentation)
- Undertaking as a community mitigates solitude in teaching

# Teaching numerical methods or teaching with numerical methods?

We can do both!

Numerical methods can create effective figures to illustrate soil mechanics concepts, but don't give the impression that numerical models are superior. Encourage critical thinking!

# Can we use numerical methods in education without teaching the underlying mathematical details?

Maybe…

Don’t ask students to «believe» to the results, but encourage curiosity.